MAT 6621 Théorie de la représentation des groupes

Hiver 2025

Plan de cours

Horaire du cours : Mondays 9:00-11:00 at AA 4186 and Tuesdays 9h00-11h00 in AA 5183. The final exam will be on April 15.

Contenu du cours:

Part 0: *Start*:
Classification problems of Linear Algebra. First examples of Representation Theory problems. Representations of abelian groups.

Part 1: *Algebras*:
Algebras over a field and modules over them. Examples. Simple modules. Jordan-Holder theorem. Schur's lemma.

Part 2: *Semisimplicity*:
Semisimple algebras. Density theorem and its consequences. Structure of a semisimple algebra over a field.

Part 3: *Representations of finite groups*:
Representations of finite groups. Maschke's theorem. Orthogonality of characters.

Part 4: *Induced representations*:
Definition and examples of induced representations. Frobenius duality.

Part 5: *Representations of the symmetric group*:
Complex representations of the symmetric group. Young diagrams and Young tableaux. Schur-Weyl duality. Schur functors.

Part 6: *Compact Lie groups*:
Examples of compact Lie groups. Maschke's theorem. Orthogonality of characters. Representations of $U(1)$ and $SU(2)$. Lie algebras and the exponential map. Complex representations of $SL(2,\mathbb{C})$.

Part 7: *Representations of classical Lie groups*:
Classification of finite-dimensional complex representations of $U(n)$ (equivalently, of holomorphic representations of $GL(n,\mathbb{C})$). Schur polynomials as characters. Generalization to other classical groups.

Références :

- *Introduction to representation theory*, by Pavel Etingof, Oleg Golberg, Sebastian Hensel, Tiankai Liu, Alex Schwendner, Dmitry Vaintrob, and Elena Yudovina. `http://www-math.mit.edu/~etingof/replect.pdf`
- W. Fulton, J. Harris, *Representation Theory: a First Course.*
- G. James, M. Liebeck, *Representations and characters of groups.*

Évaluation :

Final 40%, Homeworks 60%.

Professeur :

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