GENERATING FUNTIONCS AND APPLICATIONS

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The lectures will focus on methods and applications of generating functions to symplectic geometry and related fields. In particular we use variational capacites associated to such function to on one hand define invariants of symplectic maps and manifolds, on the other hand introduce "variational solutions" for Hamilton-Jacobi equations.

Given a Lagrangian submanifold, a generating function is a function S defined on $N \times \mathbb{R}^q$, such that

$$L = \{(x, \frac{\partial S}{\partial x}(x,\xi)) \mid \frac{\partial S}{\partial \xi}(x,\xi) = 0\}$$

We shall only be interested in those generating functions coinciding with a non degenerate quadratic form at infinity.

After preliminary results on the existence and uniqueness of such generating functions for Lagrange submanifolds, we shall give applications to the construction of symplectic invariants, and then to a number of other topics, as mentioned above.

References

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