TOPOLOGY OF ROBOT MOTION PLANNING

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Abstract. In the course we shall describe a new application of the algebraic topology in engineering. This theory allows to use the structure of the cohomology algebra of the configuration space to estimate numerically the character of instabilities appearing in the robot motion planning algorithms. We shall apply this approach to a number of specific problems which are interesting from both topological and the computer science points of view: collision free motion planning, control of a rigid body in $\mathbf{R}^3$, simultaneous control of many objects, and some others.

Algorithmic motion planning in robotics is a well established discipline; we refer to [9] for a recent survey and to [7] for a comprehensive textbook.

In general, one is given a moving system R with k degrees of freedom and a two or three-dimensional workspace V. The geometry of R and of V is given in advance which determines the configuration space of the system, X. The latter is a subset of $\mathbf{R}^k$ consisting of all placements (or configurations) of the system R, each represented by a tuple of k real parameters, such that in this placement R lies fully in V. For simplicity we may restrict our attention to a single connected component of R, the one containing a prescribed initial placement of R.

Being a subset of the Euclidean space $\mathbf{R}^k$, the configuration space X naturally inherits its *topology*. Many questions of control theory depend solely on the configuration space X viewed as a topological space. One of the advantages of this approach is that different control problems could be treated simultaneously for all systems having homeomorphic configuration spaces. It is known that any real analytic manifold can be realized as the configuration space of a simple mechanical system (linkage). Therefore topological questions of robotics lead to interesting new topological invariants of abstract manifolds.

In this course we are dealing with motion planning algorithms working as follows: the algorithm gets as its input the present and the desired states of the system and it produces as the output a continuous motion of the system from its current state to the desired state. The main purpose of the course is to explain that *the topology of the configuration space of the system imposes important restrictions on the discontinuities of the robot motion as a function of the input data.* We emphasize that these are not discontinuities of the robot motion as a function of time. The discontinuities which we study here are in the way the decision (the whole motion) depends on the input data.

To get a feeling of the problem consider the following example. Suppose that we have to teach a robot, living on an island, how to move from any given position A to any given position B. Suppose first that the island has the shape of a convex planar domain $X \subset \mathbf{R}^2$. Then we may prescribe the movement from A to B in X to be implemented along the straight line segment with a constant velocity. This rule clearly defines a motion planning algorithm which is continuous with respect to A and B. Assume now that there is a lake in the middle of the island, and that our robot is not capable of swimming and has to find its way over dry land. It is not difficult to see that in this case any motion planning strategy is discontinuous as function of the end points.

In the course I will describe a new topological approach to the motion planning problem initiated in my work [2], [3].

The course will consists of 5 lectures covering the following topics:

1. The robot motion planning problem, examples.
2. The Schwartz genus.
3. The notion of topological complexity of motion planning TC(X).
4. The order of instability of a motion planning algorithm.
5. The topological complexity of graphs, surfaces, spheres.
6. Simultaneous control of several systems.
7. Collision free motion planning of multiple objects in the space.
8. Configuration spaces of graphs and their topological complexity.
9. Random motion planning algorithms.
10. Motion planning in projective spaces and the immersion problem.

References

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