Why study number theory? Questions arise when studying any subject, sometimes fascinating questions that may be difficult to answer precisely. Number theory is the study of the most basic properties of the integers, literally taking integers apart to see how they are built, and there we find an internal beauty and coherence that encourages many of us to seek to understand more. Facts are often revealed by calculations, and then researchers seek proofs. Sometimes the proofs themselves, even more than the theorems they prove, have an elegance that is beguiling and reveal that there is so much more to understand. With good reason, Gauss called number theory the "*Queen of Mathematics*", ever mysterious, but nonetheless graciously sharing with those that find themselves interested. In this first course there is much that is accessible, while at the same time natural, easily framed, questions arise which remain open, stumping the brightest minds.

Once celebrated as one of the more abstract subjects in mathematics, today there are scores of applications of number theory in the real world, particularly to the theory and practice of computer algorithms. Best known is the use of number theory in designing cryptographic protocols (as discussed in chapter 10), hiding our secrets behind the seeming difficulty of factoring large numbers which only have large prime factors.

For some students, studying number theory is a life-changing experience: They find themselves excited to go on to penetrate more deeply, or perhaps to pursue some of the fascinating applications of the subject.