

Why give proofs? We give proofs to convince ourselves and others that our reasoning is correct. Starting from agreed upon truths, we try to derive a further truth, being explicit and precise about each step of our reasoning. A proof must be readable by people besides the author. It is a way of communicating ideas and needs to be persuasive, not just to the writer but also to a mathematically literate person who cannot obtain further clarification from the writer on any point that is unclear. It is not enough that the writer believes it; it must be clear to others. The burden of proof lies with the author.

The word “proof” can mean different things in different disciplines. In some disciplines a “proof” can be several different examples that justify a stated hypothesis, but this is inadequate in mathematics: One can have a thousand examples that work as predicted by the hypothesis, but the thousand and first might contradict it. Therefore to “prove” a theorem, one must build an incontrovertible argument up from first principles, so that the statement must be true in every case, assuming that those first principles are true.

Occasionally we give more than one proof of an important theorem, to highlight how inevitably the subject develops, as well as to give the instructor different options for how to present the material. (Few students will benefit from seeing *all* of the proofs on their first time encountering this material.)