Preface

This is a modern introduction to number theory, aimed at several different audiences: students who have little experience of university level mathematics, students who are completing an undergraduate degree in mathematics, as well as students who are completing a mathematics teaching qualification. Like most introductions to number theory, our contents are largely inspired by Gauss's *Disquisitiones Arithmeticae* (1801), though we also include many modern developments. We have gone back to Gauss to borrow several excellent examples to highlight the theory.

There are many different topics that might be included in an introductory course in number theory, and others, like the law of quadratic reciprocity, that surely must appear in any such course. The first dozen chapters of the book therefore present a "standard" course. In the *masterclass* version of this book we flesh out these topics, in copious appendices, as well as adding five additional chapters on more advanced themes. In the *introductory* version we select an appendix for each chapter that might be most useful as supplementary material.[1] A "minimal" course might focus on the first eight chapters and at least one of chapters 9 and 10.[2]

Much of modern mathematics germinated from number-theoretic seed and one of our goals is to help the student appreciate the connection between the relatively simply defined concepts in number theory and their more abstract generalizations in other courses. For example, our appendices allow us to highlight how modern algebra stems from investigations into number theory and therefore serve as an introduction to algebra (including rings, modules, ideals, Galois theory, p-adic numbers,...). These appendices can be given as additional reading, perhaps as student projects, and we point the reader to further references.

Following Gauss, we often develop examples *before* giving a formal definition and a theorem, firstly to see how the concept arises naturally, secondly to conjecture a theorem that describes an evident pattern, and thirdly to see how a proof of the theorem emerges from understanding some non-trivial examples.

[1]In the main text we occasionally refer to appendices that only appear in the *masterclass* version.
[2]Several sections might be discarded; their headings are in ***bold italics.***