

Further exploration of number theory. There is a tremendous leap in the level of mathematical knowledge required to take graduate courses in number theory, because curricula expect the student to have taken (and appreciated) several other relevant courses. This is a shame since there is so much beautiful advanced material that is easily accessible after finishing an introductory course. Moreover, it can be easier to study other courses, if one already understands their importance, rather than taking it on trust. Thus this book, *Number Theory Revealed*, is designed to lead to two subsequent books, which develop the two main thrusts of number theory research:

In *The distribution of primes: Analytic number theory revealed*, we will discuss how number theorists have sought to develop the themes of chapter 5 (as well as chapters 4 and 13). In particular we prove the prime number theorem, based on the extraordinary ideas of Riemann. This proof rests heavily on certain ideas from complex analysis, which we will outline in a way that is relevant for a good understanding of the proofs.

In *Rational points on curves: Arithmetic geometry revealed*, we look at solutions to Diophantine equations, especially those of degree two and three, extending the ideas of chapter 12 (as well as chapters 14 and 17). In particular we will prove Mordell's Theorem (developed here in special cases in chapter 17) and gain a basic understanding of modular forms, outlining some of the main steps in Wiles's proof of Fermat's Last Theorem. We avoid a deep understanding of algebraic geometry, instead proceeding by more elementary techniques and a little complex analysis (which we explain).

References. There is a list of great number theory books at the end of our book and references that are recommended for further reading at the end of many chapters and appendices. Unlike most textbooks, I have chosen to not include a reference to every result stated, nor necessarily to most relevant articles, but rather focus on a smaller number that might be accessible to the reader. Moreover, many readers are used to searching online for keywords; this works well for many themes in mathematics.⁵ However the student researching online should be warned that Wikipedia articles are often out of date, sometimes misleading, and too often poorly written. It is best to try to find relevant articles published in expository research journals, such as the *American Mathematical Monthly*,⁶ or posted at arxiv.org which is "open access", to supplement the course material.

⁵Though getting just the phrasing to find the right level of article can be challenging.

⁶Although this is behind a paywall, it can be accessed, like many journals, by logging on from most universities, which have paid subscriptions for their students and faculty.