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## Gauss's *Disquisitiones Arithmeticae*

In July 1801, Carl Friedrich Gauss published *Disquisitiones Arithmeticae*, a book on number theory, written in Latin. It had taken five years to write but was immediately recognized as a great work, both for the new ideas and its accessible presentation. Gauss was then widely considered to be the world's leading mathematician, and today we rate him as one of the three greatest in history, alongside Archimedes and Sir Isaac Newton.

The first four chapters of *Disquisitiones Arithmeticae* consist of essentially the same topics as our course today (with suitable modifications for advances made in the last two hundred years). His presentation of ideas is largely the model upon which modern mathematical writing is based. There follow several chapters on quadratic forms and then on the rudiments of what we would call Galois theory today, most importantly the constructibility of regular polygons. Finally, the publisher felt that the book was long enough, and several further chapters did not appear in the book (though Dedekind published Gauss's disorganized notes, in German, after Gauss's death).

One cannot overestimate the importance of *Disquisitiones* to the development of 19th-century mathematics. It led, besides many other things, to Dirichlet's formulation of ideals (see sections [3.19](#) [3.20](#) of appendix 3D, [12.8](#) of appendix 12A, and [12.10](#) of appendix 12B), and the exploration of the geometry of the upper half-plane (see Theorem [1.2](#) and the subsequent discussion).

As a young man, Dirichlet took his copy of *Disquisitiones* with him wherever he went. He even slept with it under his pillow. As an old man, it was his most prized possession even though it was in tatters. It was translated into French in 1807, German in 1889, Russian in 1959, English only in 1965, Spanish and Japanese in 1995, and Catalan in 1996!

*Disquisitiones* is no longer read by many people. The notation is difficult. The assumptions about what the reader knows do not fit today's reader (for example, neither linear algebra nor group theory had been formulated by the time Gauss wrote his book, although *Disquisitiones* would provide some of the motivation for developing those subjects). Yet, many of Gauss's proofs are inspiring, and some have been lost to today's literature. Moreover, although the more advanced two-thirds of *Disquisitiones* focus on binary quadratic forms and have led to many of today's developments, there are several themes there that are not central to today's research. In the fourth book in our trilogy (!), *Gauss's Disquisitiones Arithmeticae revealed*, we present a reworking of Gauss's classic, rewriting it in modern notation, in a style more accessible to the modern reader. We also give the first English version of the missing chapters, which include several surprises.