

The cover (designed by Marci Babineau and the author).

In 1675, Isaac Newton explained his extraordinary breakthroughs in physics and mathematics by claiming, “*If I have seen further it is by standing on the shoulders of Giants.*” Science has always developed this way, no more so than in the theory of numbers. Our cover represents five giants of number theory, in a fan of cards, each of whose work built upon the previous luminaries.

Modern number theory was born from PIERRE DE FERMAT’s readings of the ancient Greek texts (as discussed in section [6.1](#)) in the mid-17th century, and his enunciation of various results including his tantalizingly difficult to prove “Last Theorem.” His “Little Theorem” (chapter 7) and his understanding of sums of two squares (chapter 9) are part of the basis of the subject.

The first modern number theory book, Gauss’s *Disquisitiones Arithmeticae*, on which this book is based, was written by CARL FRIEDRICH GAUSS at the beginning of the 19th century. As a teenager, Gauss rethought many of the key ideas in number theory, especially the law of quadratic reciprocity (chapter 8) and the theory of binary quadratic forms (chapter 12), as well as inspiring our understanding of the distribution of primes (chapter 5).

Gauss’s contemporary SOPHIE GERMAIN made perhaps the first great effort to attack Fermat’s Last Theorem (her effort is discussed in appendix 7F). Developing her work inspired my own first research efforts.

SRINIVASA RAMANUJAN, born in poverty in India at the end of the 19th century, was the most talented untrained mathematician in history, producing some extraordinary results before dying at the age of 32. He was unable to satisfactorily explain many of his extraordinary insights which penetrated difficult subjects far beyond the more conventional approaches. (See appendix 12F and chapters 13, 15, and 17.) Some of his identities are still inspiring major developments today in both mathematics and physics.

ANDREW WILES sits atop our deck. His 1994 proof of Fermat’s Last Theorem built on the ideas of the previous four mentioned mathematicians and very many other “giants” besides. His great achievement is a testament to the success of science building on solid grounds.