Hints for exercises in chapter 2

Exercise 2.1.4(b). Write the integers in the congruence class $a \pmod d$ as $a+nd$ as n varies over the integers, and partition the integers n into the congruences classes mod k.

Exercise 2.1.5. Write the congruence in terms of integers and then use exercise 1.1.1(c).

Exercise 2.1.6. Write the congruence in terms of integers and then use exercise 1.1.1(e).

Exercise 2.4.1(c). Factor 1001.

Exercise 2.5.4(a). Split the integers into k blocks of m consecutive integers, and use the main idea from the first proof of Theorem 2.1. (b) Write $N = km + r$ with $0 \leq r \leq m-1$. Use (a) to get k such integers in the first km consecutive integers, and at most one in the remaining r. Compare k or $k+1$ to the result required.

Exercise 2.5.6(b). Use the results for $m = 4$ from (a). (d) Use the same idea as in (c).
(e) Study squares mod 8.

Exercise 2.5.9(b). Use that $\frac{1}{j}\binom{p-1}{j-1} = \frac{1}{p}\binom{p}{j}$.

Exercise 2.5.10(a). Treat the cases $a \geq b$ and $a < b$ separately. (b) Treat the cases $c \geq d$ and $c < d$ separately.

Exercise 2.5.13. Proceed by induction on $k \geq 1$.

Exercise 2.5.15(b). Use induction.

Exercise 2.5.16(a). Try a proof by contradiction. Start by assuming that the kth pigeonhole contains a_k letters for each k, and determine a bound on the total number of letters if each $a_k \leq 1$. (b) Use the pigeonhole principle. (c) Use induction.

Exercise 2.5.17(a). Use the pigeonhole principle on pairs $(x_r \pmod d, x_{r+1} \pmod d)$. (d) Use exercise 1.7.24.

Exercise 2.10.2. Use induction.