Hints for exercises in chapter 11

Exercise 11.2.1. If $y = 0$, then $m_1 n = n_1 m$. Now $(m, n) = (m_1, n_1) = 1$ and so $m_1 = m$ and $n_1 = n$ contradicting our construction of the pair m, n.

Exercise 11.2.5. Consecutive powerful numbers of the form $2^3 a^2$ followed by b^2, for some integers a and b.

Exercise 11.4.2. Use the product rule to compute the derivative.

Exercise 11.6.3. Given a smallest solution to $x^2 - dy^2 = 1$ expand $(x + \sqrt{d}y)^{\phi(d)}$ (mod d).

Exercise 11.6.11(c). Consider the example $1 + (2^n - 1) = 2^n$ with $m \geq 2/\epsilon$.

Exercise 11.14.3(a). Write $\begin{pmatrix} p & s \\ r & \ell \end{pmatrix} = \begin{pmatrix} a_0 & 1 \\ 1 & 0 \end{pmatrix} \cdots \begin{pmatrix} a_n & 1 \\ 1 & 0 \end{pmatrix}$ and use that $a_n \geq 2$. (b) Take determinants of the matrix equation so that $rs \equiv (-1)^n \pmod{p}$, and therefore $s = r$ or $p - r$. (c) Take the transpose of $\begin{pmatrix} p & r \\ r & \ell \end{pmatrix}$. (d) Write $\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} a_0 & 1 \\ 1 & 0 \end{pmatrix}$
$\cdots \begin{pmatrix} a_m & 1 \\ 1 & 0 \end{pmatrix}$ so that $\begin{pmatrix} p & r \\ r & \ell \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} a & c \\ b & d \end{pmatrix}$.

Exercise 11.16.2. One thought is to take 2^{a_0} if $a_0 \geq 0$ and 3^{-a_0} if $a_0 < 0$, and then use the primes 5 and 7 for a_1, etc.