

The distribution of prime numbers

5.1. Proofs that there are infinitely many primes

5.2. Distinguishing primes

5.3. Primes in certain arithmetic progressions

5.4. How many primes are there up to x ?

5.5. *Bounds on the number of primes*

5.6. *Gaps between primes*

5.7. *Formulas for primes*

5.8. Additional Exercises: Questions about primes

Appendix 5A. Bertrand's postulate and beyond

5.9. Bertrand's postulate

5.10. The theorem of Sylvester and Schur

Bonus read: A review of prime problems

5.11. Prime problems

Prime values of polynomials in one variable

Prime values of polynomials in several variables

Goldbach's conjecture and variants

Appendix 5B. An important proof of infinitely many primes

5.12. Euler's proof of the infinitude of primes

5.13. The sieve of Eratosthenes and estimates for the primes up to x

5.14. Riemann's plan for Gauss's prediction, I

Appendix 5C. What should be true about primes?

5.15. The Gauss-Cramér model for the primes

Short intervals

Twin primes

Appendix 5D. Working with Riemann's zeta-function

5.16. Riemann's plan for Gauss's prediction

5.17. Understanding the zeros

An elementary proof

5.18. Reformulations of the Riemann Hypothesis

Appendix 5E. Prime patterns:

5.19. Generalized arithmetic progressions of primes

Consecutive prime values of a polynomial

Magic squares of primes

Primes as averages

Appendix 5F. A panoply of prime proofs

5.20. Furstenberg's (point-set) topological proof

An analytic proof

A proof by irrationality

Appendix 5G. Searching for primes and prime formulas

5.21. Searching for prime formulas

5.22. Conway's prime producing machine

5.23. Ulam's spiral

5.24. Mills's formula

Appendix 5H. Dynamical systems and infinitely many primes

5.25. A simpler formulation

5.26. Different starting points

5.27. Dynamical systems and the infinitude of primes

5.28. Polynomial maps for which 0 is strictly preperiodic