Concours Putnam

Atelier de Pratique

Le jeudi, 10 septembre 14h30-15h30

1. Prove that it is impossible to choose three distinct integers a, b and c such that

$$a - b|b - c, \quad b - c|c - a, \quad c - a|a - b.$$

2. There are given N points in space (with $N > 1$). Some of them are connected to each other by lines. Prove that there are two points which are connected to the same number of points.

3. Into how many regions do n lines divide the plane, assuming no two lines are parallel and no three lines intersect in the same point?

4. How many planes are needed to cut each of the edges of the cube at least twice (intersections at a vertex don't count as intersecting an edge)?

5. Prove that there are no integers x, y and z (with $(x, y, z) \neq (0, 0, 0)$) such that

$$x^3 + 2y^3 + 4z^3 = 0.$$

6. A certain country has finitely many cities. Any pair of these cities is connected by a road. However, all roads in this country are one-way roads, and it is therefore not always possible to travel from one city to another city. Show that the country has a city ("capital") that can be reached from every other city either directly or via exactly one intermediate city.

7. Three distinct points with integer coordinates lie in the plane on a circle of radius $r > 0$. Show that two points are separated by a distance of at least $r^{1/3}$.

8. A convex pentagon has all its vertices lattice points in the plane (and no three collinear). Prove that its area is at least $5/2$.