# Concours Putnam

Atelier de Pratique

Le jeudi, 24 septembre 14h30-15h30 (Salle: AA 1411)

**Polynômes**

1. Find a polynomial with integer coefficients whose zeros include
$$\sqrt{2}+\sqrt{5}.$$

2. Let $P(x)$ be a polynomial of degree $n$. Suppose that
$$P(0), P(1), \ldots, P(n)$$
are all integers. Prove that $P(m)$ is an integer for every integer $m$.

3. The product of two of the four zeros of the quartic equation
$$x^4 - 18x^3 + kx^2 + 200x - 1984 = 0$$
is $-32$. Find $k$.

4. Let $a, b, c$ be distinct integers. Can the polynomial
$$(x-a)(x-b)(x-c) - 1$$
be factored into the product of two nonconstant polynomials with integer coefficients?

5. Let $P(x) \in \mathbb{Z}[x]$ have degree $n > 5$ and $n$ distinct integer roots. Prove that $P(x) + 3$ also has $n$ distinct real roots.

6. Let $f(x)$ be a polynomial with real coefficients, and suppose that
$$f(x) + f'(x) > 0$$
for all real $x$. Prove that $f(x) > 0$ for all real $x$.

7. Let $k \geq 6$ and let $P(x) \in \mathbb{Z}[x]$. Suppose that at $k$ distinct integer arguments, all the values of $P$ belong to
$$\{1, 2, \ldots, k-1\}.$$
Prove that all these $k$ values are equal.

8. Two players $A$ and $B$ play the following game. $A$ thinks of a polynomial with nonnegative integer coefficients. $B$ must guess the polynomial. $B$ has two shots: she can pick a number and ask $A$ to return the polynomial value there, and then she has one more such try. Can $B$ win the game?

9. If $a, b, c > 0$, is it possible that each of the polynomials
$$P(x) = ax^2 + bx + c, \qquad Q(x) = cx^2 + ax + b, \qquad R(x) = bx^2 + cx + a$$
has two real roots?