Concours Putnam

Atelier de Pratique
Le jeudi, 7 novembre 12h30-13h30
5448 Pav. André Aisenstadt

Théorie des nombres

Euler Totient Function The Euler totient function $\phi(n)$, denoting the number of positive integers not exceeding n and relatively prime to n is given by

$$\phi(n) = n \prod_{p_i|n} \left(1 - \frac{1}{p_i}\right)$$

where the p_i's are prime numbers.

Euler's Theorem If a and n are relatively prime integers, then $a^{\phi(n)} \equiv 1 \mod n$.

Pythagorean Triples All relatively prime positive integer solutions to $x^2 + y^2 = z^2$ with x odd and y even are of the form $x = u^2 - v^2$, $y = 2uv$, $z = u^2 + v^2$.

1. Let p_n be the n^{th} prime number. Show that the sequence $\{q_n\}$ defined by $q_n = p_{n+1} - p_n$ is unbounded.
2. Show that the product of four consecutive positive integers is never a perfect square.
3. Find the last two digits of 3^{2011}.
4. How many positive integers divide at least one of 10^{40} and 20^{30}?
5. Show that for any positive integer r, we can find integers a, b such that $a^2 - b^2 = r^3$.
6. Find all solutions to $1! + 2! + 3! + \ldots + n! = m^2$ in positive integers.
7. Let $a > 1$. Show that $a^n + 1$ is prime only if a is even and $n = 2^k$.
8. Which members of the sequence 101, 10101, 1010101, ... are prime?
9. The number 2^{333} has 101 digits, and begins with 1. How many of the numbers in the set $2, 4, 8, 16, ..., 2^{333}$ begin with 4?