Concours Putnam

Atelier de Pratique

Le mardi, 21 novembre 12h30-13h30

5448 Pav. André Aisenstadt

Récurrences

1. Let $a_0 = 1$, $a_1 = \frac{3}{5}$, $a_{n+1} = \frac{6}{5}a_n - a_{n-1}$. Show that $|a_n| \leq 1$ for all n.
2. Solve $a_{n+1} = \sqrt{a_n a_{n-1}}$ where $0 < a_0 < a_1$ and find $\lim_{n\to\infty} a_n$.
3. Prove that the sequence $a_0 = 2, a_1 = 3, a_2 = 6, a_3 = 14, a_4 = 40, a_5 = 152, a_6 = 784$, ... with general term $a_n = (n+4)a_{n-1} - 4na_{n-2} + (4n-8)a_{n-3}$ is the sum of two well-known sequences.
4. The sequence a_n of non-zero reals satisfies $a_n^2 - a_{n-1}a_{n+1} = 1$ for $n \geq 1$. Prove that there exists a real number α such that $a_{n+1} = \alpha a_n - a_{n-1}$ for $n \geq 1$.
5. Find
$$\lim_{n\to\infty} (2+\sqrt{2})^n - \lfloor (2+\sqrt{2})^n \rfloor$$
where $\lfloor x \rfloor$ is the largest integers $\leq x$.
6. Solve
$$f(n+1) = 1 + \sum_{i=0}^{n-1} f(i)$$
with $f(0) = 1$.
7. Solve
$$y_n(1 + ay_{n-1}) = 1.$$
8. Given $a_n = (n^2+1)3^n$, find a recurrence relation $a_n + pa_{n+1} + qa_{n+2} + ra_{n+3} = 0$. Hence evaluate $\sum_{n=0}^{\infty} a_n x^n$.
9. The sequence a_n is defined by $a_1 = 2, a_{n+1} = a_n^2 - a_n + 1$. Show that any pair of values in the sequence are relatively prime and that $\sum \frac{1}{a_n} = 1$.