Concours Putnam

Atelier de Pratique
Le mardi, 14 novembre 12h30-13h30
5448 Pav. André Aisenstadt

Théorie des nombres

1. Show that the sum of two consecutive primes is never twice a prime.

2. Prove that two consecutive Fibonacci numbers are always relatively prime.

3. Show that there exist 1999 consecutive numbers, each of which is divisible by the cube of an integer.

4. Find all non-negative integral solutions $(n_1, n_2, \ldots, n_{14})$ to
$$n_1^4 + n_2^4 + \ldots + n_{14}^4 = 1599.$$

5. a) Do there exist 2 irrational numbers a and b greater than 1 such that $\lfloor a^m \rfloor \neq \lfloor b^n \rfloor$ for every positive integers m, n?

   b) Do there exist 2 irrational numbers a and b greater than 1 such that $\lfloor am \rfloor \neq \lfloor bn \rfloor$ for every positive integers m, n?

6. Suppose $n > 1$ is an integer. Show that $n^4 + 4^n$ is not prime.

7. Prove that there are no primes in the following infinite sequence of numbers:
$$1001, 1001001, 1001001001, 1001001001001, \ldots$$

8. Let n be a positive integer. Suppose that 2^n and 5^n begin with the same digit. What is the digit?

9. Prove that if n is an integer greater than 1, then n does not divide $2^n - 1$.