Concours Putnam

Atelier de Pratique
Le mardi, 7 novembre 12h30-13h30
5448 Pav. André Aisenstadt

Géométrie

1. Let A, B, C, D be four points in space forming a quadrilateral. Show that the midpoints of AB, BC, CD, DA form a parallellogram.
2. Let A, B, C, D be four points on a plane. Prove that $|AB||CD|+|BC||AD| \geq |AC||BD|$.
3. Let $v_1, \ldots, v_k$ be k vectors in the plane of norm $|v_i| \leq 1$. Prove that there exists a choice of signs such that $v = \pm v_1 \ldots \pm v_k$ satisfies $|v| \leq \sqrt{2}$.
4. Let $P_1, \ldots, P_n$ be points on the unit sphere. Prove that $\sum_{i \leq j} |P_i P_j|^2 \leq n^2$.
5. Does the circle $S^1 = \{x^2 + y^2 = 1\}$ in the plane contain a closed subset that contains exactly one of each pair of diametrically opposite points?
6. Show that there are no 7 lines in the plane such that there are at least 6 points in the plane which lie on the intersection of just three of the lines and at least 4 points in the plane which lie on the intersection of just two of the lines.