

Concours Putnam

Atelier de Pratique

Le mercredi, 18 septembre 12h30-13h30

La salle 5448 Pav. André Aisenstad

1. Gryffindor fans tell the truth when Gryffindor wins and lie when it loses. Fans of Hufflepuff, Ravenclaw, and Slytherin behave similarly. After two matches of quidditch with the participation of these four teams (with no draws and each team playing exactly one game), among the wizards who watched the broadcast, 500 answered positively to the question “Do you support Gryffindor?”, 600 answered positively to the question “Do you support Hufflepuff?”, 300 answered positively to the question “Do you support Ravenclaw?”, and 200 answered positively to the question “Do you support Slytherin?”. How many wizards support each of the teams? Note: Each wizard is fan of exactly one of the teams.
2. Into how many regions do n lines divide the plane, assuming no two lines are parallel and no three lines intersect in the same point?
3. Find at least one positive integer that is at least 3 times smaller than the sum of its positive proper divisors.
4. A certain country has finitely many cities. Any pair of these cities is connected by a road. However, all roads in this country are one-way roads, and it is therefore not always possible to travel from one city to another city. Show that the country has a city (“capital”) that can be reached from every other city either directly or via exactly one intermediate city.
5. Three distinct points with integer coordinates lie in the plane on a circle of radius $r > 0$. Show that two points are separated by a distance of at least $r^{1/3}$.
6. Find the digit before the decimal point and the first two digits after the decimal point in the decimal notation of $(1 + \sqrt{2})^{2024}$.