Concours Putnam

Atelier de Pratique

Le jeudi, 28 novembre 12h30-13h30

La salle 5448 Pav. André Aisenstad

Géométrie

1. Let A, B, C, D be four points in space forming a quadrilateral. Show that the midpoints of AB, BC, CD, DA form a parallellogram.
2. Is it possible to place a finite number of parabolas on a plane so that their interiors cover the entire plane?
3. Let A, B, C, D be four points on a plane. Prove that $|AB||CD|+|BC||AD| \geq |AC||BD|$.
4. There are n red and n blue points on a plane, no three of which lie on the same line. Prove that it is possible to draw n segments with different-colored endpoints that have no common points.
5. Let $P_1, \ldots, P_n$ be points on the unit sphere. Prove that $\sum_{i \leq j} |P_i P_j|^2 \leq n^2$.
6. The plane is divided into equilateral triangles with side 1 by three infinite series of equally spaced parallel lines. M is the set of all their vertices. A and B are two vertices of one triangle. It is allowed to rotate the plane by 120° around any of the vertices of the set M. Is it possible to transform point A into point B using several such transformations?
7. Does the circle $S^1 = \{x^2 + y^2 = 1\}$ in the plane contain a closed subset that contains exactly one of each pair of diametrically opposite points?
8. Show that there are no 7 lines in the plane such that there are at least 6 points in the plane which lie on the intersection of just three of the lines and at least 4 points in the plane which lie on the intersection of just two of the lines.