MAT 6229: Special topics in PDEs

Topics in geometric spectral theory

Part 1: Preliminaries

1.1 Strings, drums and the Laplacian.

1.2 The Laplace-Beltrami operator a Riemannian manifold. Examples: flat tori and round spheres.

1.3 Sobolev spaces. Weak spectral theorems for the Laplacian.

1.4 Elliptic regularity and strong spectral theorems.

Part 2: Variational principles and their applications

2.1 Variational principles. Domain monotonicity and Dirichlet-Neumann bracketing.

2.2 Inequalities between the Dirichlet and Neumann eigenvalues for Euclidean domains.

2.3 Weyl's law and Pólya's conjecture. Berezin-Li-Yau inequalities.

Part 3: Nodal geometry of eigenfunctions

3.1 Chladni's plates. Courant's nodal domain theorem and its applications.

3.2 Geometric properties of nodal sets. Density of the nodal sets, lower bound on the size of the nodal set in dimension two. Yau's conjecture.

3.3 Nodal sets on surfaces and eigenvalue multiplicity bounds.

Part 4: Geometric inequalities for eigenvalues

4.1 The Faber–Krahn inequality. Symmetrization.

4.2 Cheeger's inequality. The first eigenvalue and the inradius of planar domains.

4.3 The Szegő–Weinberger inequality for the first Neumann eigenvalue.

4.4 Hersch's theorem for the first eigenvalue on the sphere. Topological upper bounds for eigenvalues on surfaces.

Part 5: Heat equation, spectral invariants and isospectrality

5.1 Heat equation on a Riemannian manifold. Heat kernel asymptotics and spectral invariants. Weyl's law on Riemannian manifolds.

5.2 Isospectral manifolds: Milnor's example.

5.3 Sunada's construction.

5.4. "Can one hear the shape of a drum?" Transplantation of eigenfunctions. Examples of spectral rigidity.

Part 6: The Steklov Problem and the Dirichlet-to-Neumann map

6.1 Steklov eigenvalue problem. Isoperimetric inequalities for Steklov eigenvalues. Weinstock's inequality.

6.2 The Dirichlet-to-Neumann map and the boundary Laplacian. Hörmander-Pohozhaev identities. Weyl's law for the Steklov spectrum.