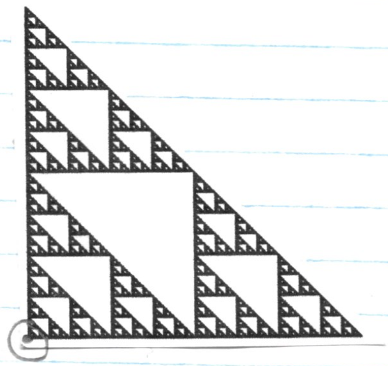
TP4

7 février 2025

Retour sur le laboratoire

1.



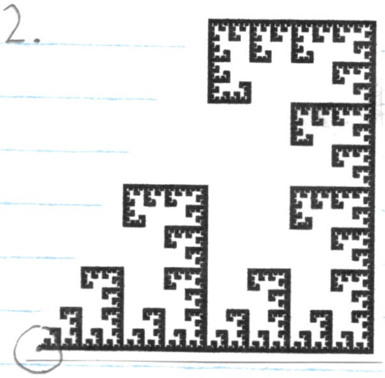
origine

$$T_1\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1/2 & 0\\0 & 1/2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$$

$$T_2\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1/2 & 0\\0 & 1/2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}1/2\\0\end{pmatrix}$$

$$T_3\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1/2 & 0\\0 & 1/2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}0\\1/2\end{pmatrix}$$

2.



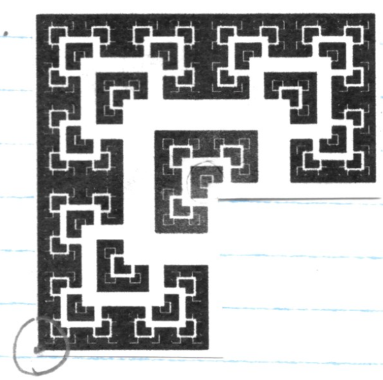
origine

$$T_1\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1/2 & 0\\0 & 1/2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$$

$$T_2\begin{pmatrix}x\\y\end{pmatrix} = 1/2\begin{pmatrix}\cos(\frac{\pi}{2}) & -\sin(\pi/2)\\\sin(\frac{\pi}{2}) & \cos(\pi/2)\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}1\\1/2\end{pmatrix}$$

$$T_3\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1/2 & 0\\0 & 1/2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}1/2\\0\end{pmatrix}$$

3.



origine

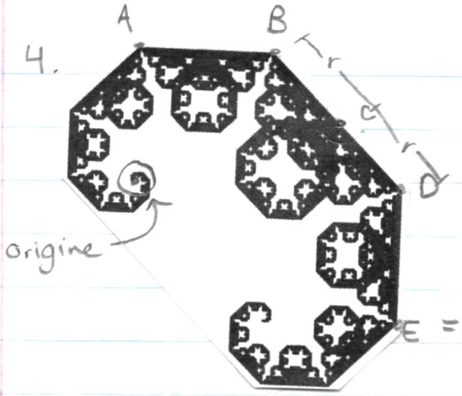
$$T_1\begin{pmatrix}x\\y\end{pmatrix} = 1/2\begin{pmatrix}\cos(\pi/2) & -\sin(\pi/2)\\\sin(\pi/2) & \cos(\pi/2)\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}1/2\\0\end{pmatrix}$$

$$T_2\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}1/2 & 0\\0 & 1/2\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}0\\1/2\end{pmatrix}$$

$$T_3\begin{pmatrix}x\\y\end{pmatrix} = 1/2\begin{pmatrix}\cos(-\pi/2) & -\sin(-\pi/2)\\\sin(\pi/2) & \cos(\pi/2)\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}1/2\\1\end{pmatrix}$$

$$T_4\begin{pmatrix}x\\y\end{pmatrix} = \begin{pmatrix}\frac{3}{10} & 0\\0 & \frac{3}{10}\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}7/20\\7/20\end{pmatrix}$$

4.



Supp. que long(AB) = 1

$$T_1\begin{pmatrix}x\\y\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\cos(\pi/4) & -\sin(\pi/4)\\ \sin(\pi/4) & \cos(\pi/4)\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix}$$

$$T_2\begin{pmatrix}x\\y\end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix}\cos(-\pi/4) & -\sin(-\pi/4)\\ \sin(-\pi/4) & \cos(-\pi/4)\end{pmatrix}\begin{pmatrix}x\\y\end{pmatrix} + \begin{pmatrix}1\\0\end{pmatrix}$$

$T_1(BD) = AB$

$\Rightarrow \text{long}(AB) = r\,\text{long}(BD) = 1$

$T_2(BD) = DE$

$\Rightarrow \text{long}(DE) = r\,\text{long}(BD) = 1$

$BD = T_1(DE) \cup T_2(AB)$

$= BC \cup CD$

$\Rightarrow \text{long}(BD) = \text{long}(BC) + \text{long}(CD)$

$= r + r$

$= 2r$

$\Rightarrow \text{long}(AB) = r\,\text{long}(BD)$

$\Leftrightarrow 1 = 2r^2$

$\Leftrightarrow r = 1/\sqrt{2}$

Calcul des dimensions

1. $N(1) = 1$

$N(1/2) = 3$

$N(1/4) = 9$

$\vdots$

$N(1/2^n) = 3^n$

$$D = \lim_{n\to\infty} \frac{\log(N(1/2^n))}{\log(1/(1/2^n))}$$

$$= \lim_{n\to\infty} \frac{\log(3^n)}{\log(2^n)} = \lim_{n\to\infty} \frac{n\log(3)}{n\log(2)} = \frac{\log(3)}{\log(2)}$$

2. Identique au 1.

3. On utilise la formule $r_1^D + r_2^D + \quad + r_m^D = 1$.

On a $\left(\frac{3}{10}\right)^D + \left(\frac{1}{2}\right)^D + \left(\frac{1}{2}\right)^D + \left(\frac{1}{2}\right)^D = 1$

$\Leftrightarrow \left(\frac{3}{10}\right)^D + 3\left(\frac{1}{2}\right)^D - 1 = 0$

On utilise la méthode de la bissectrice.

Considérons $f(D) = \left(\frac{3}{10}\right)^D + 3\left(\frac{1}{2}\right)^D - 1$. On cherche pour quel D est-ce que $f(D) = 0$.

$f(0) = \left(\frac{3}{10}\right)^0 + 3\left(\frac{1}{2}\right)^0 - 1 = 3$

$f(2) = \left(\frac{3}{10}\right)^2 + 3\left(\frac{1}{2}\right)^2 - 1 = \frac{9}{100} + \frac{3}{8} - 1$

$= \frac{72 + 300}{800} - 1$

< 0

point milieu de $[0,2]$: 1

$\hookrightarrow f(1) = \frac{3}{10} + \frac{3}{2} - 1 = \frac{8}{10} > 0$

point milieu de $[1,2]$: $1/2$

$\hookrightarrow f(3/2) \approx 0{,}22 > 0$

point milieu de $[3/2, 2]$: $7/4$

$\hookrightarrow f(7/4) \approx 0{,}0135 > 0$

point milieu de $[7/4, 2]$: $15/8$

$\hookrightarrow f(15/8) \approx -0{,}075 < 0$

point milieu de $[7/4, 15/8]$: $29/16$

$\hookrightarrow f(29/16) \approx -0{,}033$

On a donc que $D \in (7/4, 29/16)$, $D \approx 1{,}78$