Homework 3
Math 16110-Section 50

Due: Tuesday October 22th

Exercise 1. Let E and F be two sets and $f : E \to F$. Show that the two following propositions are equivalent

(i) For all subsets A and B of E,
$$f(A \cap B) = \emptyset \Longrightarrow f(A) \cap f(B) = \emptyset.$$

(ii) f is injective.

Exercise 2. Let $f : E \to F$. Show that f is bijective if and only if, for all $A \subset E$ we have $f(E \setminus A) = F \setminus f(A)$.

Exercise 3. We define a relation $\mathcal{R}$ on $\mathbb{R}^2$ by

$$(x, y)\mathcal{R}(x', y') \Longleftrightarrow \text{There exist } a, b > 0 \text{ such that } x' = ax \text{ and } y' = by.$$

1. Show that $\mathcal{R}$ is a relation.

2. Give the equivalence classes of $A = (1, 0)$, $B = (0, -1)$, $C = (1, 1)$.

3. Give all the equivalence classes of the relation $\mathcal{R}$.

Exercise 4. Let E be a set and $A \subset E$. We recall the definition of the symmetric difference Δ, for $B, C \subset E$,

$$B\Delta C = (B \setminus C) \cup (C \setminus B).$$

We define a relation $\mathcal{R}$ on $\mathcal{P}(E)$ by
$$B\mathcal{R}C \Longleftrightarrow B\Delta C \subset A.$$

1. Show that $\mathcal{R}$ is a relation.

2. Let $B \subset E$. Show that the equivalence class of B is

$$[B] = \{(B \cap (E \setminus A)) \cup K;\ K \subset A\}$$

Exercise 5. Let E be a non-empty set and $\mathcal{A} \subset \mathcal{P}(E)$ such that

$$\text{For all } X, Y \in \mathcal{A}, \text{ there exists } Z \in \mathcal{A},\ Z \subset (X \cap Y).$$

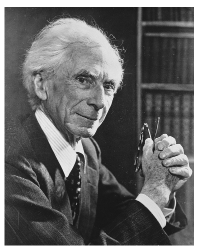
We define the relation $\mathcal{R}$ on $\mathcal{P}(E)$ by

$$B\mathcal{R}C \Longleftrightarrow \text{There exists } X \in \mathcal{A},\ X \cap B = X \cap C.$$

1. Show that $\mathcal{R}$ is a relation.

2. Find the equivalence classes of $\emptyset$ and E.

Gottlob Frege



Bertrand Russel