Homework 5
Math 20400-Section 51

Due: Monday February 10th

Exercise 1. **1.** Use a (well-chosen) quadratic approximation to compute, up to a small error,

$$e^{\sin(3.16)\cos(0.02)}.$$

2. Use an affine approximation to compute, up to a small error,

$$\arctan\left(\sqrt{4.03} - 2e^{0.01}\right)$$

Exercise 2. Find the equation of the tangent plane for each of the surfaces below at the given point (x_0, y_0, z_0).

1. $\mathscr{S}_1 = \{(x, y, z) \in \mathbb{R}^3 \mid z = \sqrt{19 - x^2 - y^2}\}$ at $(x_0, y_0, z_0) = (1, 3, 3)$.

2. $\mathscr{S}_2 = \{(x, y, z) \in \mathbb{R}^3 \mid z = \sin(\pi xy)e^{2x^2y-1}\}$ at $(x_0, y_0, z_0) = (1, \frac{1}{2}, 1)$.

Exercise 3. Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by $f(x, y) = x^2 - 2y^3$.

1. Find the equation of the tangent plane $\mathscr{P}_{M_0}$ to the surface $\mathscr{S}\{(x, y, z) \in \mathbb{R}^3 \mid z = f(x, y)\}$ at any regular point $M_0 \in \mathscr{S}$.

2. For the point $M_0 = (2, 1, 2) \in \mathscr{S}$, find all points $M \in \mathscr{S}$ such that the tangent plane at M is parallel to $\mathscr{P}_{M_0}$.

Exercise 4. We consider the following surface in $\mathbb{R}^3$:

$$\mathscr{S} = \{(x, y, z) \in \mathbb{R}^3 \mid 2x^2 - 3xy + y + 2z^2 = 1\}$$

1. Prove that $\mathscr{S}$ is a smooth hypersurface.

2. Describe geometrically the set of points $M \in \mathscr{S}$ such that the tangent plane at M contains the point $(0, 0, 0)$.

Exercise 5. We consider the following surface $\mathscr{S}$ and line $\mathscr{L}$

$$\mathscr{S} = \{(x, y, z) \in \mathbb{R}^3 \mid x^2 + y^2 - z^2 = 1\} \quad \text{and} \quad \mathscr{L} = \left\{(x, y, z) \in \mathbb{R}^3 \mid x = 1 \text{ and } y = z + 2\right\}$$

1. Show that $\mathscr{S}$ is a smooth hypersurface.

2. Find all tangent plane(s) of $\mathscr{S}$ which contains the line $\mathscr{L}$.

Brook Taylor
(1685–1731)

Pierre de Fermat
(160X–1665)