MATH SCIENCES INVESTIGATION (MSI): THE ANATOMY OF INTEGERS AND PERMUTATIONS

Jennifer and Andrew Granville

INT. PRECINCT CONFERENCE ROOM - DAY

Detective Jack von Neumann, (50), well dressed, though with the look of a man who has had too much good wine and is weary from too many dead bodies, addresses a room full of junior detectives and uniformed officers.

VON NEUMANN
Thank you all for making it in for this presentation. As you know, we’ve had no good leads with these two homicides, so we brought in Professor Gauss and his team from the Bronx Math Institute to look over the evidence. Professor Gauss told me that they’ve come up with some surprising new connections, and asked us to meet today to hear about them from his new assistant, Mr. Langer. Mr. Langer, please.

He indicates at Sergei Langer (30), a blond, muscular student with a disfiguring scar (shaped like $\int$) across his cheek, to come to the front of the room, from the chair at the side where he is sitting. Langer gets up with his laptop, looking a little nervous.

Cuts to a card on the screen reading THREE WEEKS EARLIER.

INT. HI-TECH LECTURE HALL, BRONX INSTITUTE FOR MATH – DAY
(THREE WEEKS EARLIER)

Professor Gauss, (45), tall, dark and handsome with a confidence in his ability that gives him an attractive aura of being ‘easy in his skin’, is lecturing to a class of eager YOUNG MATH STUDENTS. At the back of the lecture hall, a DOCUMENTARY FILM CREW are filming.
Gauss
Welcome to all of you. Forensic science - one of the most exciting areas of modern math. You’ve all completed your pre-reqs, first year graduate math training ... now let’s be sure we’re all on the same page. So, some definitions. ANATOMY?

The students are startled at his barking of the word.

Gauss (cont’d)
Come on, come on. No time to dawdle. Someone, anyone, definition of anatomy?

Langer whispers into the ear of the beautiful STUDENT next to him, who sits in front of an open laptop. She recoils at his hot, damp breath.

Langer
The scientific study of the shape and structure of an organism and the inter-relation of its various parts...
This is too elementary to bother with.

He squeezes her thigh.

Langer
This.... now this is what I call ‘anatomy’.

The student, Emmy Germain (23), dark haired and too beautiful to be in a class that promises to cut up cadavers, squirms with disgust.

Emmy
Get off me.

Still no public response to Gauss’s question.

Gauss (cont’d)
Best answer gets to be my assistant on my next case.

Oh, the students get it. Now, a sea of hands shoot up, including Langer’s. As Gauss looks in their direction, Langer goes to answer but Gauss cuts him off, pointing to Emmy.

Gauss
You, the young woman with the laptop.

Emmy hesitates, shoots Langer a look.
**Emmy**
The scientific study of the shape and structure of an organism and the inter-relation of its various parts...

She gains confidence as Gauss nods approvingly. Langer is irritated.

**Emmy (cont’d)**
The art of separating the parts of an organism in order to ascertain their position, relations, structure, and function.

**Gauss**
Excellent. A perfect definition. And can you consolidate your success with a definition of ‘forensic’?

Emmy hesitates again.

**Gauss (cont’d)**
No time for false modesty. Do you know or don’t you?

Langer, who’s been silently fuming, cannot contain himself any longer. He leaps up.

**Langer**
Relating to the use of science or technology in the investigation and establishment of facts or evidence... I am your rightful assistant, Professor Gauss.

**Gauss**
Please be seated.

Langer sits, flushed and irritated.

**Gauss (cont’d)**
I do not believe I asked you, Mr...?

**Langer**
Langer.

**Gauss (cont’d)**
Langer. Nonetheless, Mr. Langer is correct. So we are now clear on what we are doing, and we can begin...

Gauss thinks for a moment.
Gauss (cont’d)
My new case will involve a lot of hard work, so for now I will take you both...

Gauss indicates at Emmy and Langer,

Gauss (cont’d)
...on as my assistants. But, be warned, I do not typically take on more than one assistant at a time, so you are going to need to impress me to be kept on.

Ext. Steps of Bronx Institute for Math - Day
A small group of students, including Emmy, are clustered at the top of the steps. The director of the documentary crew, Barry Bell, is talking to the camera. Langer is jostling to stand next to Emmy and in the eye of the camera.

Barry
And now we are going to talk to Professor Gauss’s new assistant, Emmy??...Emmy - tell us a bit about yourself, your background - where did you do your undergrad?

Emmy
Well...I....it’s a bit....

Langer interrupts.

Langer
Don’t push her on this - she didn’t graduate.

Emmy is blushing, embarrassed

Langer
However, I am also going to work as Professor Gauss’s assistant....I studied in Paris and Princeton....

Barry is intrigued by Emmy’s confusion.

Barry
Emmy? You don’t have a degree? So how’d you get into grad school, studying under Professor Gauss?
EMMY
Well, it’s - I ...I studied on my own, my parents didn’t want me to go into forensics - no business for a girl they said - but I worked on my own and I managed to find Professor Gauss’s email address and I wrote to him with some of my ideas, and then he accepted me onto his course.

LANGER
I, on the other hand, graduated summa cum......

The documentary cameraman abruptly moves the camera away from Langer to pick up Gauss as he pushes his way through the glass doors. His car draws up, followed by a police car. Detective von Neumann, climbs out, looking a little disheveled.

VON NEUMANN
Hey, Professor Gauss - can I talk to you for a minute?

GAUSS
Good morning, Newman. What can I do for you?

VON NEUMANN
We’ve got another. Same M.O.

Langer strains to hear the conversation.

GAUSS
And?

von Neumann looks shifty.

VON NEUMANN
I guess we need you to come in and take a look.

Gauss smiles. He might be looking a tiny bit triumphant.

INT. MORGUE - DAY

Gauss, Emmy, Langer, von Neumann and SEVERAL ATTENDANTS, all dressed in white coats, masks and gloves, are standing above a BURLY MALE DEAD BODY lying on a slab. The documentary crew are still shooting as Barry talks quietly into a microphone.
Barry

We have been here for the past three hours. The team has worked without a break, led by the famous Dr. Gauss, assisted by his two pupils...Detective Newman has agreed to brief us -

He turns to von Neumann.

Barry (cont’d)

What have you discovered so far, sir?

Von Neumann

He’s got a bar code tattooed on his neck - we ran it and turns out it’s his social security number - name’s Arnie Integer.

Barry

Arnie Integer - the politician?

Von Neumann

I am not in a position to confirm or deny that.

Gauss is straightening up, peeling off his gloves.

Gauss

So - what can you see?

He throws the question to a startled Emmy, whose eyes widen above her mask.

Emmy

He’s - er - well, he’s....

Langer

Dead? Yeah, we can all see that.

Gauss

You need to break this habit of speaking when you aren’t being spoken to, Mr Langer.

Gauss looks penetratingly at Langer, and then turns back to Emmy.

Gauss

So - how was he killed?
EMMY
He seems to have been knifed, with a cross cut into his chest with some sort of serrated instrument.

Langer sniggers and mutters under his breath.

LANGER
You do like to state the obvious.

GAUSS
Mr Langer. “The world is full of obvious things which nobody by any chance ever observes.”

Gauss turns suddenly to the camera and pulls off his mask. The BOOM OPERATOR clumsily moves the microphone to above his head.

Barry is excited and starts whispering instructions to the cameraman.

GAUSS (CONT’D)
So, you may be asking why I’m here. A Professor of Forensic Science - why was I called in when this looks like a standard murder? Slightly bizarre, with the cross cut into the chest - but nevertheless no stranger than 100 murders committed every year.

Barry nods encouragingly.

GAUSS (CONT’D)
This is the second murder with the exact same method. The cross cut in the chest, the victim being left to bleed to death...

He reaches his hand into the wound...

GAUSS (CONT’D)
...the heart removed surgically.

VON NEUMANN
He...the perp...he must have left something behind. There must be a direct link between the two murders, and we need the best forensics to find it. Some DNA has got to be in there somewhere, something that’ll take us to him.

GAUSS
If it’s there, we’ll find it.
INT. MORGUE - LATER

The body has been completely dissected. Langer is being violently sick.

EMMY
Total decomposition. Total.

Gauss goes to remove his gloves. Emmy still stands with her hands in the cadaver’s stomach. von Neumann takes a surreptitious drink from his hip flask.

GAUSS
Feel again.

EMMY
What for?

GAUSS
You’ll find it. Mr. Integer here is full of them.

Emmy is startled. She pulls out a slimy, but solid organ.

EMMY
Oh. It’s a prime, sir. It hasn’t decomposed, and there are more of them dispersed around Integer’s body.

GAUSS
Exactly.

He turns to an ORDERLY.

GAUSS CONT’D
Bring me the paperwork on that last one...

VON NEUMANN
Permu Ta Tion?

Gauss nods.

VON NEUMANN
I can tell you all you need to know - ain’t gonna help though. Apart from the knife wound, no similarities.

EMMY
There has to be, surely?
VON NEUMANN
Nope - It’s obvious - look at him. Integer was Caucasian, 6’ 5”, 60 years old, looks like he lived hard. Permu Ta Tion, she was French-Oriental, delicate little thing: Five foot, a ballet dancer.

GAUSS
It may look obvious. But, to paraphrase that sublime detective, Sherlock Holmes, we cannot assume anything. To solve this case it is necessary that we should be able to utilize all the facts. Admittedly, this implies, a possession of all the facts, which, even in these days of computers, is unlikely.

Langer, still wiping his mouth and looking pretty green, snickers.

LANGER
Sherlock Holmes? A fictional amateur detective? The bottom line, Professor, is that Integer and Permutation are about as similar as apples and i-pods.

Von Neumann gives Langer a startled, curious look. The orderly returns and hands Gauss a sheaf of papers.

GAUSS
But, Mr. Langer, have you ever dissected an apple and an i-pod?

Emmy shoots Gauss a questioning look. Langer just throws up again.

INT. GAUSS’S OFFICE - DAY
The room is large and light and elegantly furnished. A slim computer sits on an antique desk, a mahogany drinks cabinet stands beneath an original Matisse.
Emmy and Langer are sitting on a soft, cream leather couch, reading through the papers given to Gauss by the orderly. Lying next to them is a violin. Gauss is leaning over a billiard table, cue in hand, taking aim. Langer slams the papers down.

LANGER
Nothing. No similarities.

EMMY
Except...
CLICK. Gauss hits a ball. Langer looks up anxiously - what has she spotted that he hasn’t?

Gauss

Yes.

Emmy

It’s not a similarity really. But, in both Integer and Permu Ta Tion, the internal organs were completely decomposed. Except in Integer there was a smattering of primes and in Permu Ta Tion a smattering of cycles.

CLICK. Another ball is hit.

Gauss

Good. You noticed.

Langer scowls.

Langer

Yes, but where does that get us?

Emmy

I guess it would make sense to calibrate the primes with the cycles... But how? (she shakes her head). Langer is right - they’re about as different as apples and i-pods.

Gauss rubs some chalk onto his cue.

Gauss

Of course he’s right in that he’s identified the problem. But that doesn’t mean it’s unsolvable. (He chuckles) “On the contrary, my dear Watson, you can see everything. You fail, however, to reason from what you see”.

Langer

Not Holmes again.

Gauss

Think about it. Do you know how rare primes are?

Emmy shakes her head. Gauss is taking aim, lining up his cue with the balls.

Gauss (cont’d)

How about cycles?
Emmy
No...

Langer
And where would it get us even if she did?

Now Gauss starts to rapidly hit the balls, with sharp, expert CLICKS.

Gauss
We know that roughly one out of every \( \log x \) integers up to \( x \) is prime (CLICK), and that exactly one in every \( N \) permutations on \( N \) letters is a cycle (CLICK), so we could try to calibrate by replacing \( N \) (CLICK) when we measure the anatomy of a permutation with \( \log x \) (CLICK) when we measure the anatomy of an integer.

Gauss continues cleaning up the table. Emmy grins, excited.

Emmy
So then we’ll have the calibration we need to start comparing Integer and Permutation’s cycles and prime factors. But...what do we compare?

Langer
It’s obvious. We need to go back to the paperwork...

Emmy wrinkles her face.

Emmy
How will that help us?

Langer
If there is something to be found that is where it will be. You can’t underestimate the value of the well organized, well documented file. You should always have the facts at your fingertips.

Gauss
Perhaps...but more importantly, you need to have the imagination to extract the unknown from the known, the unwritten from the written.

Neither Emmy or Langer answer.
More to the point - you need to compare how many cycles there were in Permutation to how many primes were found inside Integer.

Emmy starts to leaf through the papers again.

...and see whether they were laid out in the same way.

He pots the black.

So where do we start comparing members of the Permutation family with people from the Integer cartel? We can start with what everyone knows: A typical Permutation on $N$ letters has about $\log N$ disjoint cycles. If our hypothesized ‘calibration’ is to make sense then we should be able to simply swap out the ‘$N$’ for a ‘$\log x$’, and guess that a typical Integer has about $\log \log x$ distinct prime factors. Sounds unlikely, but it is true - one of the classic investigations that is in every training manual - tracing the exploits of that unbeatable combination of English solidity and Eastern brilliance, detectives Hardy and Ramanujan.

There is BUZZ of excited whispering - the students all know the legend of that dynamic duo. Langer is looking particularly sour today.

Excuse me, Dr. Gauss. This is meant to be a class for modern mathematical forensics at the graduate level - haven’t you got anything a little more up-to-date, a little more challenging, a little more abstract?

I am afraid if you are looking for a little more abstraction you have entered the institute a little too late - Professor Joe Ten-Dick would have been an ideal mentor for you, in your quest for the perfect abstraction, had he not left us for....well, for self-imposed exile. His methods were brilliant but I always felt that they needed to be a little more grounded in the concrete.

Langer is pressing his fist against his scar, his eyes screwed up in pain.
I don’t know what Professor Ten-Dick has to do with.... Well, I am not too interested in what a typical permutation or integer has or doesn’t have. What about the more extreme cases? Those are the interesting ones. Do you know anything about those permutations with more cycles than usual? Do you know if there are many?

You ask good questions, Langer. But let’s see if you know some good answers. Here’s one from the pre-reqs. Tell me what a Bell curve is.

Data that seems chaotic often organizes itself into certain recognizable patterns. The most common is where, when you graph the data, the plot is shaped like a bell around the average.

And are there different bell curves?

All the bells have the same basic shape, though the center may appear in different places, and some may be fatter than others. We measure the width of the bell by its ‘variance’.

Good.

Gauss is obviously impressed. Langer looks pleased with himself, his scar throbbing with pleasure.

And here is the thing: If one examines all members of the permutation family which act on $N$ letters, one finds that the number of cycles each possess seems random but, when you do the count, those numbers are distributed like a bell curve, with average log $N$ and variance also log $N$. What happens when we calibrate again, replacing $N$ by log $x$ to guess what happens with the integers?
It was a rhetorical question but Langer can’t stop himself interrupting. He leaps up.

**LANGER**
It’s not a guess ... fifty years ago Detectives Erdős and Kac showed that it’s more-or-less TRUE for the number of prime factors of integers up to $x$, average and variance both being around $\log \log x$.

Gauss stares him down. Langer flushes with embarrassment and sits.

**GAUSS**
Thank you once again for your help, Mr. Langer. I hardly know where I would be without you.

Gauss reaches below his desk and rummages around.

**GAUSS**
Aaargh, I’m not going to find it. Do you remember why else I mentioned Detective Kac?

**EMMY (QUICKLY)**
His report, “Can you hear the shape of a drum?”

**GAUSS**
Exactly! Someone was listening...only two weeks ago I mentioned that this should be required reading for all first-year forensic math students, explaining how an effect can help you determine all the possible causes. Good work. See you next time.

Langer looks down miserably at his desk.
Peeping out from beneath Langer’s papers is an original reprint of “Can you hear the shape of a drum?”

**INT. FRONT OF LECTURE HALL, BRONX INSTITUTE OF MATHEMATICS - DAY**
Most of the students are leaving the room. A few keen students are at the front asking further questions, until the last ones left are Emmy and Langer.

**EMMY**
OK, I know that was for our benefit. We know how many components there are as we look over all the members of the populations, but how does that help us with the lay out? I don’t get it.

Gauss shoots a piercing look at Langer.
I don’t know either.

Well, I have my suspicions, but I don’t want to prejudice your investigation by sharing my hunches. Sorry Mr. Langer, but to quote Sherlock Holmes again - “It is a capital mistake to theorize before one has data. One begins to twist facts to suit theories, instead of theories to suit facts.” Besides, I want to see which one of you can be the first to figure this out.

Langer’s face lights up. He figures it’s his chance to outwit Emmy.

A competition?

You could call it that, that is if you want to be the one to present our findings to the police and maybe become my student ... I’ll start you off - I’ll help you figure out how you are going to conduct the autopsies. Knowing what you are looking for from the start can make things go a lot more smoothly. Follow me. I feel the need for a cold marble slab coming on.

Langer looks queasy.

CLOSE UP on a wonderful selection of runny goat’s cheeses and blue cream cheese, laid out on a cold marble slab.

The ATTENDANT offers samples to Gauss, Emmy and Langer. Gauss and Emmy enthusiastically taste the cheese, but Langer refuses it. He looks more queasy than before.

I don’t eat dairy.

Gauss is startled.

Really? How strange...(he is lost in thought for a moment). You know, I’ve been asking myself, if we have log\(N\) cycles in a typical \(N\) letter permutation, then how do we study the lengths of those cycles? What is the right way to do it? You know ‘Occam’s razor’, right?
LANGER
The simplest explanation that fits the facts should be the correct one.

GAUSS
Exactly. (He points at another cheese) And what is the simplest sequence that has about \( \log N \) elements up to \( N \)?

The attendant passes him a selection of cheeses on a plate and some crackers.

EMMY
I guess the most regular one would be something like \( 1, e, e^2 \) and so on, all the way up to \( e^{\log N} \).

LANGER
But that can’t be right? The cycle lengths couldn’t always be that regular, could they?

Gauss sits at a small table and starts to butter a cracker.

GAUSS
I doubt it, but the sequence you’ve come up with suggests a way to proceed - if you take the log of each element of your sequence then you get \( 1, 2, 3 \), and so on.

He slices off some blue cheese.

GAUSS (cont.)
So why don’t we take the log of the length of each cycle inside Permutation, and see how those numbers are distributed?

EMMY
Why would we do that?

GAUSS
Well, there are about \( \log N \) such numbers, each lying between 0 and \( \log N \), so I wonder how they would be distributed. Will they all be clumped in one spot?

Gauss spreads the blue cheese onto the buttery cracker.
**Gauss**

Will they be well spread out? I dunno. I want you to do the analysis, and work out what is the right way to describe their lay out. That is, if I am not talking complete nonsense.

He chews on the cracker....turns to Langer.

**Gauss**

Sure you won’t have some, Langer? Some well ripened goat’s cheese is just what you need inside you before you start cutting up cadavers again.

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**Ext. Morgue - Evening**

Langer and Emmy climb out of a cab. Barry jumps out of the van that he and the camera crew have been camped out in. He marches towards them.

**Barry**

Hey guys, where have you been? We need to follow all your moves.

The taxi, with Gauss in it, speeds off. Barry calls after him.

**Barry**

Hey! Wait prof....damn it. I feel like he’s avoiding us.

Emmy and Langer exchange a look.

**Langer**

Yeah, could be. Hey, you can come in and interview me if you like. Emmy and I are back to work on the cadavers.

He goes into the morgue and Barry is about to follow him but Emmy stops him. She can’t resist the tiniest element of flirtation entering her voice

**Emmy**

Hey - I don’t suppose you guys could give me a ride...

**Barry**

Well...where are you going?
EMMY
I have to get home to feed my dog - if you could give me a ride there and back I can get back here quicker ... I don’t want Langer to get ahead...we are really stuck right now, and he might find the clue that could move us ahead....

CUT TO:

INT. DOC CREW VAN — NIGHT
The cameraman and sound recordist are in the back of the van, Barry is driving and Emmy is in front in the passenger seat.

BARRY
You’re right, I didn’t understand a word of what the Prof said, but I also got the impression that he left a lot unsaid for you two to figure out.

EMMY
Very astute. This could be a thesis project. I guess he wants it to be all our own, but I’m a little scared that I’ve got it all wrong and I don’t know who to talk to.

BARRY
Well, don’t let Langer know your doubts. Explain to me how you’re going to go about it. I might not understand much but I can nod my head convincingly.

Emmy laughs. They are on the freeway and pass two SUVs and a convertible.

CAMERAMAN
Man, that is weird, we just passed those guys five minutes ago.

BARRY
Shit...you’re right. That is so random.

Emmy sits bolt upright.

EMMY
What did you say?

CAMERAMAN
I said - it was weird that we passed ....
Emmy
No, Barry. He said ‘random’...that passing those cars was ‘random’. That’s it!

She leans over and kisses Barry on the cheek. He blushes.

Cameraman
What’s going on?

Emmy
I think I have it....switch the camera on and I’ll explain.

The sound recordist swings the boom over Emmy’s head and the camera blinks on.

Emmy
Okay. The problem we have is to give a good description of the distribution of the lengths of the various cycles in Permu....

Barry
In Permu? But you’ve already examined each cycle that came out of that cadaver and noted its length.

Emmy
Right. So if we take any member of Per Mu Tation’s family, not just our cadaver, but any old Jane Rho, the cycle lengths are all between 1 and $N$. That means that the logarithms of these lengths must all be between 0 and $\log N$, and there are roughly $\log N$ of these numbers. So the question is to determine how they are spread out. Got it so far?

Barry nods his head unconvincingly.

Barry
But what I’m not getting is what this has got to do with ‘random’.

Emmy
Patience. When we took our class in traffic control, we learnt that, after a while, cars on a freeway spread out ‘randomly’.
Cameraman
Well this traffic isn’t equally spaced. Look at it. I’ve never seen equally spaced out traffic in all the years I’ve been driving.

Emmy
You’re right. Being ‘randomly spread out’ doesn’t mean that they are equally spaced out, it means that they are spaced according to a more complicated concept called a ‘Poisson point process’. I’m not sure it applies to Permu’s case, but what if I did all the tests to determine whether the logs of the lengths of the cycles in Permu - a typical case - are spread out like a ‘Poisson point process’, like cars on a freeway?

Barry HONKS his horn at the two SUVs and the convertible as he overtakes them.

Emmy
No, really I mean it. And then I’d want to see if the loglogs of the prime factors of typical integers are the same, or whether they’re laid out in a different way.

Barry
And do you also have a theory for why crap drivers always travel in threes, and mostly drive SUVs?

Emmy
No, listen....it makes sense. I mean it. And then I’d want to see if the loglogs of the prime factors of Arnie Integer - also typical of his kind - are the same, or whether they’re laid out in a different way.

There is silence in the van.

Emmy
Guess you think I’m crazy, huh?

Barry (nodding his head)
Trust me.

Emmy
Yeah, you’re probably right ... and if that is your reaction, then what will Professor Gauss’s be? He’ll probably put me back with the rest of the class and Langer will solve the case and become Gauss’s new assistant.

Emmy screams out loud in frustration, causing Barry to swerve.
Barry
What was that? Some kind of weird math yell?

Emmy
I’m sorry. I just get so frustrated having to be controlled and logical all the time. It’s so hard...

She pounds the dashboard.

Emmy (contd.)
I’ve got so much to prove - turn the damn camera off.

Barry
No way - this is the drama we’re looking for....(he softens) Hey, cool it. Haven’t you given me an interview telling me how ole K.F. is like the greatest math guy there ever was? Did he get that way without taking a few risks, and sounding a little crazy from time-to-time? If that’s your best shot, you should go for it.

Emmy is cheered up by Barry’s attitude. Barry is now more focussed on the road as the two SUVs and the convertible overtake them again.

Barry
But I’m here to tell you - there’s nothing random about this. This is war.

He speeds up, HONKING his horn.

Int. Morgue - Day
The two bodies, Integer and Permu Ta Tion, are laid on slabs side by side. Emmy stands examining some carefully arranged organs that lie on tables next to each body. Langer is studying his laptop screen.

Emmy
Identical! They’re identical. I don’t believe it.

The cameraman who is working on a sudoku puzzle, leaps up and switches on the camera.

Cameraman
What? Show us what you’ve got.

Barry grabs his mike and starts talking to camera.
Down in the morgue that has become home to these two young investigators, a breakthrough seems to have occurred.

He turns to Langer.

CAMERAMAN
Hey Langer, aren’t you interested? Seems to be some sort of breakthrough going on?

Langer beckons to the camera to come nearer, carefully turns off his computer, packs it away. He has to move a paperback copy of ‘The Hound of the Baskervilles’ to make a space for the computer. He licks his lips and prepares to enjoy his moment of glory.

LANGER
It’s no more than I expected ... I think you will find that Emmy has discovered either a poisson or a normal distribution? These are pretty well ubiquitous in such situations.

Emmy’s face looks ashen, staring disbelievingly at Langer.

LANGER (cont’d)
Perhaps Emmy would like to stop and remember forensic probability theory, 1-0-1, before getting quite so excited? Of course, if she ever read my emails, she would know that I’d cced her on an email to Professor Gauss, proposing exactly this idea.

Emmy flushes red as the cameraman moves in closer, sensing drama.

EMMY
I told you that I wasn’t going to read my email while I am in the lab so that I could get some work done. Besides, you send out ten a day, for god’s sake, to everyone on your ‘list’; how can you expect anybody to read them all? You knew what I was looking for...couldn’t you have just told me?

Langer looks very pleased with himself and taps his briefcase.

LANGER
While you’ve been out feeding your dog and studying the traffic, I’ve been doing some serious research. See you tomorrow ... I’ve earned a good night’s sleep, and probably the right to become the professor’s student.
He leaves, smug and satisfied. There is a horrible silence. Emmy is stunned.

**BARRY**

Emmy - are you alright?

Emmy looks doubtfully at the bloody organs lined up on the two gurneys.

**EMMY**

Well, I guess it’s true that the cycle lengths and prime factor sizes have to be distributed somehow - so perhaps it was obvious that it would be something random... so ...

She moves over to the dissected corpse of Integer and studies his body thoughtfully. She suddenly seems to gain strength.

**EMMY (cont’d)**

...So, to get something interesting, we need to look at *unusual* aspects of the anatomies of Per Mu Ta Tion and Integer that are much less likely to be identical!

She starts to pull on her rubber gloves.

**BARRY**

But it’s midnight...

**EMMY**

And unlike Langer, I haven’t earned a good night’s sleep.

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**INT. PRECINCT CONFERENCE ROOM - DAY**

A card on the screen reads: **TWO WEEKS LATER**

von Neumann stands at the lectern.

**VON NEUMANN**

So let us hear from Mr. Langer who has been working closely with Professor Gauss.

He indicates Langer who has a new moustache and looks like he has dressed up in a geeky kind of way. He seems to have won the competition.

**DISSOLVE**

Langer stands at the lectern indicating at his **Power Point** slides.
Langer

Any permutation on $N$ letters is made up of indecomposable cycles, say of lengths $d_1 \leq d_2 \leq \cdots \leq d_\ell$ say. These add up to $N$, of course, so all these cycle lengths must be between 1 and $N$. A typical member of the permutation family has about $\log N$ cycles, so we needed to analyze the roughly $\log N$ numbers $\log d_1 \leq \cdots \leq \log d_\ell$, each of which are between 0 and $\log N$. Notice that the average spacing between consecutive $\log d_j$’s is about 1. So how are the $\log d_j$’s distributed? Our team has run all the tests and shown that they look like a Poisson point process: In other words the proportion of randomly chosen intervals of length $\lambda$ which contain exactly $k$ values $\log d_j$ is about

$$e^{-\lambda \lambda^k \over k!}.$$  

And what about integers? We calibrate by replacing $N$ by $\log x$, so for a typical integer up to $x$ we have about $\log \log x$ prime factors $p_1, p_2, \ldots, p_\ell$. When we ran the tests we found that the numbers $\log \log p_1, \cdots, \log \log p_\ell$ analogously form a Poisson point process in the interval between 0 and $\log \log x$. So we found that this aspect of the anatomies of permutations and of integers appears to be the same.

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INT. MORGUE - SEVERAL HOURS LATER (TWO WEEKS EARLIER)

Emmy is sitting, head in hands, bloody gloves discarded on the floor beside her. The cameraman is filming her. Barry is dozing.

EMMY

Can you please turn that thing off?

CAMERAMAN

Sorry, it’s just that your despair will make compelling viewing.

Barry jerks awake.
EMMY
Thanks a bunch, that really makes me feel better. I’ve failed utterly to find anything else. I have been trying to compare the smallest cycles and prime factors and I can’t do it, it doesn’t make any sense. And Langer will be back any minute, after a good night’s sleep, showered and refreshed, ready to face the Prof and I’ve got nothing - nothing...and Langer will probably have a case all neatly ready to present to Neumann.

BARRY
What doesn’t make sense? I’m not following...by way of a change.

EMMY
Put the damned camera down and I’ll tell you...

Barry nods at the cameraman who gratefully switches off the camera and slumps down.

CAMERAMAN
In words of one syllable?

EMMY
Whatever.

Emmy crosses to the bloody pile of decomposed organs piled on the gurney beside each body and is instantly focused. Barry looks queasy.

CAMERAMAN
Shouldn’t you put your gloves on?

EMMY
Huh? Oh...yeah, thanks.

She absentmindedly pulls on the gloves and holds up a long, flexible, circular tube-like mass.

EMMY (CONT’D)
So, here is the smallest cycle Permu Ta Tion had in her. Tiny right?

CAMERAMAN
What is it?

Emmy is getting exasperated.
EMMY
I said - it’s a cycle....And I am supposed to compare it with this?

She holds up a small, chunky wodge of muscle.

EMMY (cont’d)
The smallest prime factor from good old Arnie Integer here. And these are supposed to be representative of whichever type of cadaver they come from.

Barry looks at the two organs.

BARRY
It’s impossible.

EMMY
Of course it’s impossible. How can I compare the smallest primes to the smallest cycles, they’re just such different objects?

BARRY
Yeah!

The door to the outside opens and Langer walks in. Freshly showered and dressed. Emmy ignores him and goes on talking.

EMMY
And, besides, what chance is there that two integers are gonna have the same smallest prime factor if it is not super-tiny? More-or-less none, as far as I can tell, so the calibration which professor Gauss proposed can’t work, can it?

BARRY
No?

LANGER
No, of course not! So I see Emmy has finally figured it out. That it doesn’t work.

Emmy looks surprised.
Langer (cont’d)
I went home last night when I realised it was impossible, that you can’t calibrate them because of the smallest elements. The great Gauss himself - sending us off on a wild goose chase based on a hunch. He should take the advice of his fictional hero “Detection is, or ought to be, an exact science and should be treated in the same cold and unemotional manner” - and that sure doesn’t mean hunches.

Int. Precinct conference room - Day
Langer continues his Power Point presentation.

Langer
Part of the art of Forensic Mathematics is to decide what evidence to ignore, if any. In this investigation we found that the smallest components cannot be sensibly compared, but only have a tiny influence on the key comparisons, so we will ignore these peripheral effects.

Langer takes a sip of water. Furtively glances at the audience as he sips to gauge how much they seem to be with him. Looks satisfied.

Langer
We created our calibration by comparing what proportion of integers are prime with what proportion of permutations are cycles. But what if we now compare the proportion of integers that have exactly two prime factors, with the proportion of permutations that have exactly two cycles — will they be the same? Or three? Or more? It turns out that in each case the formulas for these proportions are complicated, involving several different types of factors. But, when we replace $N$ in the formulas for permutations, by $\log x$ we do obtain the formulas for integers. Quite a co-incidence!

Int. Morgue - Morning
Emmy is sitting, head in hands, in despair.

Langer
I only said that the great Gauss himself had got it wrong, there’s no need to go into mourning.
EMMY
Why didn’t you say something? I’ve been working on this all night. I’m exhausted.

Langer shrugs and smiles.

LANGER
Wanted to be fresh for the prof this morning....you’re looking a little rough I must say.

Emmy glares at him but is stopped from telling him what she thinks of him by the double doors SLAMMING open as Gauss strides in, clutching a tray with coffees, and a paper bag.

The cameraman switches his camera on again and Barry jumps up.

GAUSS
So, have we cracked it yet? Donuts? Coffee - soy latte, caramel frapaccino, mmm. I love the smell of formaldehyde in the morning.

He hands out coffees all round and sits down expectantly.

LANGER
Emmy has been here all night Professor. I know she’ll have a lot to say.

Emmy was about to take a sip of her coffee, then stops, removes her bloodied gloves and sits, despairing.

EMMY
I’m sorry professor. I can’t do it. I can’t even figure out what it is I have to compare. Either the smallest component is too small to compare between the organisms. Or, if we only look at those in which the smallest component is a little larger and thus comparable, it occurs too rarely to observe.

Gauss pulls a china plate and silver knife from his briefcase and delicately dissects a croissant.

GAUSS
You’re right ... So let’s go back to the problem. And take a broader perspective.

LANGER
A broader perspective? I thought that was where you and Professor Ten-Dick parted company?

Gauss is puzzled.
Gauss
And where did you get that idea from?

Langer is now evasive, his fist held tightly against his scar.

Langer
I....well...a friend of mine, Count Nicholas Bourbaki....he told me...

The documentary crew are suddenly alert and they move toward Langer.

Barry Bell
Mr. Langer, who is this Count Bourbaki? He sounds very exotic.

Langer
Oh, no one, he....

Gauss laughs congenially.

Gauss
Go ahead, Langer, tell the assembled company about your friend Bourbaki - I’d like to hear who introduced you. Joe Ten-Dick?

Langer’s eyes are darting all over the room. He is cornered.

Langer
No, no. I have never met Professor Ten-Dick. You know he vanished long ago.

Gauss smiles a secret smile.

Barry Bell
But a Count? How did you come in contact with a Count? Do they still exist in this day and age?

Langer bristles at this imagined slight.

Langer
Count Bourbaki certainly exists. I met him in the Cafe Flores.

Barry Bell
In Paris?
LANGER
Yes, when I was studying there. Myself and some others....we would meet and talk....but he would not want me to discuss him - especially not on the television. He always prefers to stay anonymous, however brilliant his ideas are. And they are brilliant...I really won’t talk about him anymore. Turn that damned camera off.

BARRY BELL
I’m sorry Mr. Langer. You’ve always been keen to talk to us before, but if you insist.

The camera crew move away from Langer who is now fiercely rubbing his scar. Gauss has been ruminating.

GAUSS
Well Mr. Langer, your ‘friend’, Nicholas Bourbaki, got it wrong as did Joe Ten-Dik. What we want is a fundamental description of the respective anatomies. Right?

EMMY
Right.

She is weary with frustration. Gauss looks at her kindly.

GAUSS
“You see but you do not observe.”

He finishes his croissant and prepares to leave.

GAUSS (CONT’D)
Instead of specifying the smallest component, we could look at all of the integers, and permutations, that have no components smaller than a given size?

LANGER
How is that different?

Now Emmy laughs out loud with pleasure. She can’t resist answering.

EMMY
Because, at a stroke, we have not only got rid of the issue that the smallest component happens rarely, but we can also work in the range where we know our organisms are comparable.

Emmy is grinning at the elegance of Gauss’s argument.
Gauss

Very well put, my dear.

Langer is frowning, pressing his fist against his scar again.

Langer

That’s genius, professor.

Gauss stares at Langer for a moment.

Gauss

“They say that genius is an infinite capacity for taking pains. It’s a very bad definition, but it does apply to detective work.”

Langer looks triumphant, the pain in his scar forgotten.

Langer

A Study in Scarlet!

Gauss is startled.

Gauss

Yes, Mr. Langer. Yes. Now let’s hope that you can apply what you have learned from both me and from Mr. Sherlock Holmes!

Int. Morgue - Night

Emmy is working furiously, alternating between computer screen and cadavers, obviously graphing data she is obtaining. She looks like she has been working pretty solidly for a couple of days, tired but elated.

Gauss enters, in evening dress, slamming into the room, carrying three coffees in hand. He is followed by a very fed up von Neumann.

Gauss starts speaking before he is through the door.

Gauss

So what was it that couldn’t wait? So urgent that you had to disturb my enjoyment of a rare performance of ‘Leider de Onne Worter’? You found something?

Emmy is going to take her time responding; it’s her show now and she knows it. Gauss hands her a coffee, and looks around.
Where is everyone?

Emmy shrugs.

They’d had enough. Langer offered to take them on a tour of the building - a backstage view of the morgue. He leapt at the chance to get away from the computer - he thinks we’re on a wild goose chase.

von Neumann perks up at this information.

So he is here - with the film crew.

Yes.

Well, Professor Gauss, I will leave you to your investigations and discovery. I have something to attend to.

Gauss is irritated.

You needn’t have left the concert if you aren’t going to stay and hear what Emmy has to say.

Each to his own, Professor, each to his own.

Von Neumann leaves and Gauss looks at Emmy questioningly. She takes a deep breath.

I caught the goose...

Gauss looks at her expectantly.

Thanks for the coffee, sir. I can’t tell you how much I needed that. Your intuition was right, dead on in fact, though I haven’t quite got it all figured out.
Gauss nods, willing her on. Emmy takes another sip of her coffee.

**Emmy (cont’d)**

Well, thinking about how to set up what you suggested, I counted integers \( n \), for which \( \log \log p \geq \frac{1}{10} \log \log n \) for every prime factor \( p \) of \( n \). Then I counted the permutations in which \( \log C \geq \frac{1}{10} \log \sigma \) for every cycle length \( C \) in \( \sigma \). And you’ll never guess what? They were in exactly the same proportion in each population.

**Gauss**

That’s good. That’s very good. (He is getting more excited.) And presumably you get the same thing happening when you replace a tenth by any fraction you like?

**Emmy**

Right, at least as far as I can measure.

**Gauss**

So what exactly is the formula for the proportion?

**Emmy**

Well I can compute the proportion for any value you start with in place of a tenth, but I haven’t been able to figure out a simple formula for the function.

**Gauss**

What do you mean you can’t find a simple formula for the function? Have you tried all the usual tests? All the usual software?

**Emmy**

Yes, and no luck. It decays to its limit really fast, super fast, so there is hardly enough resolution on my screen to graph it beyond a certain point.

Gauss looks suspicious.

**Gauss**

Let me have a look at that graph.

Gauss moves around to look at the monitor by Emmy’s side. He starts sucking on his coffee as he looks, eyes fixed on the screen.
Gauss
Well I never. I haven’t seen a function like that since my days in sieve theory. I’ll bet that’s Buchstab’s function. You never heard of Buchstab’s function, huh?

Emmy shakes her head “no”. Gauss starts tapping on the keyboard.

Gauss
No I guess not. In our courses we tend to show you functions that can be easily described, not the one’s that only have a transcendental definition. Well, Buchstab’s function is something that can only really be usefully described as a certain average of its history. It is self-referential, and so hard to compute.

Gauss finishes tapping on the keyboard, and steps back.

Gauss (cont’d)
There, I’ve pulled it up. Look they’re identical. Amazing. You’ll never guess where I got this particular graph of Buchstab’s function from ... It comes up in modelling brains - after all, what is in your brain other than a suitably weighted average of its history? So I got a good picture of the function from their website. This is great, Emmy; we couldn’t have asked for a more persuasive result.

Emmy
What do you mean? Why is this so good?

Gauss
Well, look at all of the results you got before. They were to be expected. Statistics of the anatomies of integers and of permutations come in certain well-known patterns. But Buchstab’s function? No one expects to see that. And both populations have this for their distribution function? No one can believe that it is a co-incidence, surely?

Gauss thinks for a minute. Evidently experiences some doubt about what he just said.

Gauss
So can we be even more persuasive? What other tests can we try that might show other unexpected similarities?
Gauss walks off in his own little world, chin in hand, thinking. Emmy meanwhile looks very pleased with herself and goes straight to the keyboard. Gives him a few seconds to himself.

**EMMY**

Can you take a look at this, please, professor?

Emmy stands smiling expectantly, full of confidence, at the screen. Gauss takes a minute to come out of his thoughts to look at the monitor.

**GAUSS**

What’s that?

**EMMY**

Since the graphs that you asked for went so well, I thought maybe I would compare the smallest two prime factors with the smallest two cycles, and three, and four, etc.

**GAUSS**

Brilliant! I would have got there in a minute but you already thought of it. And what did you find? Were they all the same?

Emmy nods happily. Gauss looks. Eyes widen as he stares at the monitor, turns and grabs Emmy by either shoulder.

**GAUSS**

Oh, that’s fantastic. I have never seen anything quite like these functions. We’ll have to analyze them further, though I bet they’ll turn out to be sieve functions also.

Emmy has gone quiet, though still smiling. She is more than happy with the reception she has received so far but she has left the best till last.

**EMMY**

There’s more.

**GAUSS**

More. What more can there be?

**EMMY**

Well, I figured that since we compared the organisms from each population with no components smaller than a given size, and got such a good match, why not compare the organisms from each population with no components larger than a given size?
Good point. And?

Same thing. Identical functions arise, very fast decay, and, again, I was unable to identify them. A different function from last time. You want to see the pictures?

What do you think?

Emmy taps on the keyboard. Gauss can hardly contain himself.

It’s the Dickman-de Bruijn function. I’m sure of it. You see that function all the time in advanced cryptography when you are trying to figure out how fast different algorithms run. Did you take that class yet?

No.

You should do. It’s a beautiful subject, even though it’s practical. Anyway, the Dickman-de Bruijn function. What do you know. It is also a transcendental function. No simple definition; can only be usefully described as a certain average of its history. This is great. These can’t be simple co-incidences.

Gauss turns to Emmy

You have done a truly remarkable job here. I am convinced by this excellent labwork, and by your ideas that translate so well into practice. You certainly have enough for a good thesis, perhaps even good enough for one of my students.

Emmy knew that what she had done was good, but had no idea that it was this good.

So now I am going to have to convince von Neumann, though I’m not sure, even with all of this evidence and all of the proofs that I think you will be able to give, that we will have enough to satisfy him.
As he starts to leave the room, Langer enters followed by the camera crew.

**LANGER**
And here we are back in the .....oh, good evening Professor. What are you doing here?

**GAUSS**
It seems that Emmy has solved our case.

Langer can barely speak - his jaw has literally dropped.

**LANGER**
But that’s impossible. She knows who the murderer is?

**GAUSS**
No, that was not our case, that is Newman’s case. No, what Emmy has proved is that our two victims are, for all intents and purposes, twins, anatomically speaking.

Langer’s face changes, he is looking almost pleased. Neither Gauss or Emmy notice.

**LANGER**
But that’s impossible too.

Gauss smiles wickedly.

**GAUSS**
And how often have I said to you that when you have eliminated the impossible, whatever remains, however improbable, must be the truth?
Langer stops and sips his water, smiles at the audience. He is about to get to something controversial.

Langer

Similar remarks apply when comparing the proportion of permutations whose cycles have length $> N/u$ to the proportion of integers up to $x$, all of whose prime factors are $> x^{1/u}$.

That our two populations should display these features in common, both being described by these enormously complicated functions, is very surprising, and should make us suspicious that this is something more than mere co-incidence.

There is some noise in the room. What he has just said is provocative - he expected a reaction and he got it. He smiles patronizingly at the audience.
INT. MORGUE - DAY

Langer sits slumped on a chair, ignored, as the camera crew shoot close ups of Emmy’s delighted face.

EMMY
Did you really just say that you would take me on as a student?

GAUSS
I did - I also said that I don’t know how I’m going to persuade von Neumann of what we’ve got.

INT. POLICE PRECINCT - DAY

Emmy walks along the corridor towards an office with a sign which reads ‘Superintendent’ on the door. A heated argument is evidently taking place inside. She hesitates and listens.

VON NEUMANN (O.S.)
You expect me to believe this madness? You think the mayor is going to buy this? How am I going to get elected with a case that says a female illegal immigrant Asian gymnast and a male Caucasian bodybuilding film star turned politician were not only killed by the same serial killer but were in fact twins?

GAUSS
Calm down Newman - calm down - you don’t spend enough time out on the street, out there, talking to the people. Shut up here in your office all day.

Emmy takes a deep breath and knocks.

VON NEUMANN (O.S.)
Come.

Emmy goes in.
INT. VON NEUMANN’S OFFICE - CONTINUOUS

Both von Neumann and Gauss ignore Emmy as she slides into the room.

VON NEUMANN
What have ‘the people’ got to do with it?

GAUSS
Credit them with some intelligence. Show them the evidence...

von Neumann bangs on the table in frustration.

VON NEUMANN
That’s just the point. I don’t have it. Look, the bottom line is that I think I have a perp for Permu-Ta-Tion’s murder - looks cast iron. But I don’t have anything except your crazy theories to tie him to Integer’s murder - You haven’t convinced me, so how can I convince a jury? What you’ve told me so far could apply to any corpse. These two are typical of their types....

GAUSS
What do you mean by that? I’m not quite sure what you are getting at, von Neumann?

VON NEUMANN
Integer and Ta Tion are typical of their types...

Emmy mumbles under her breath.

EMMY
As typical as any asian gymnast or body builder film star politician...

VON NEUMANN
I need more. Bring me more, Gauss, or go back to your ivory tower and teach ‘calculus’ as pure forensics - stop trying to get down and dirty. Leave the applied mathematics to the people who can handle it.
EXT. CITY RECORDS OFFICE - DAY
Gauss, Langer and Emmy stand on the steps of the library.

GAUSS
Okay. You guys know what you are looking for?

EMMY
Yes, the autopsy reports, and more specifically the largest component, for any Permutation with exactly $\ell$ cycles and all Integers with exactly $\ell$ prime factors, in particular for atypical values of $\ell$.

GAUSS
Good girl. Should be a cinch.

Emmy looks doubtful.

LANGER
What about me?

GAUSS
Oh. Mr. Langer. You can present our initial findings to the boys in blue.

LANGER
And what about you. Where are you going?

GAUSS
To look for ‘why’.

EMMY
To the ‘Y’?

GAUSS
No. Why?
Langer points to his final Power Point slide. He is winding down.

Langer
How large is the largest component of an integer or a permutation if it has more components than is usual? Perhaps if there are more components than usual then there is more chance to have a particularly large one? Or perhaps, since their average size is smaller, the largest component is smaller than is typical? In fact the largest cycle of such a permutation is almost always smaller than for a typical permutation, and can be understood by another complicated function. As you might expect by now, the analogous “complicated function” describes the analogous situation for integers: that is for the largest prime factor of those integers which have rather more prime factors than is typical.

Exactly the same, and exactly the same again when we look at those members of each population with fewer components than usual. It seems that no matter what statistic we look at, the two yield the same proportions. Could it be a coincidence? With so much evidence, it seems increasingly unlikely that it could be, and we surely must suspect that there is an underlying cause.

Int. Basement of Morgue – Records office – Day
The room is a vast, vaulted tomb of a room, with dripping ceiling and peeling walls. Emmy is perched on a packing case, surrounded by open files - reading.
INT. PRECINCT CONFERENCE ROOM - DAY

Gauss sits in the corner of the room, reading a book entitled ‘Who got Einstein’s office?’ Langer is just finishing up his PowerPoint presentation to a room full of uniformed and non-uniformed POLICE OFFICERS.

LANGER
So, in every test that we did, the usual ones and some we invented because we didn’t quite believe what we were seeing, we found that there was no significant anatomical differences between the two populations, permutations and integers.

There is a palpable silence for several beats. And even more beats.

INT. LABORATORY – DAY

Gauss is looking through his microscope.

INT. BASEMENT OF MORGUE – RECORDS OFFICE – DAY

Emmy jumps up in excitement, tears two pages out of a file and dashes out.

INT. PRECINCT CONFERENCE ROOM - DAY

VON NEUMANN
So you are telling me that the forensic evidence is incontrovertible. That these two bodies are essentially identical twins? This 6’ 5”, 60 year old, looks like he lived hard, Caucasian, and this delicate little five foot French Oriental ballet dancer?

LANGER
Right

EXT. FINITE FIELDS - DAY

Emmy pulls up at the entrance to a huge expanse of green, surrounded by fencing. There is a double set of gates with a sign above reading FINITE FIELDS, spelt out in fancy ironwork.

Constantly referring to the papers that she tore out of the file, Emmy walks to a barbecue pit with a spit suspended over it.
She refers to the papers again. She can’t help herself. She is violently sick into the pit.

**INT. PRECINCT CONFERENCE ROOM - DAY**

**VON NEUMANN**
And more than that, you find exactly the same proportions of any specific anatomy type in each of these two populations, no matter how peculiar these precise anatomical characteristics?

**LANGER**
That’s about the size of it, Detective.

**VON NEUMANN**
Well, I’m not convinced.

The door BURSTS open and Emmy appears.

**EMMY**
I think I have the very thing that will convince you, detective.

**LANGER**
There’s no more you can say...I’ve explained it all.

**EMMY**
Not this you haven’t.

She quickly and efficiently hooks up her her computer and starts a ‘slide show’.

**EMMY**
I knew there had to be something. One more thing that would convince you. It had to be there...

On the screen body parts are coming up in succession. A hand. Another hand. A foot. Each one burned and charred and labelled. Langer is looking queasy.

**EMMY**
Ms Polly Nomial. Found dead three years ago. She had been dismembered and various body parts found spread over Finite Fields. But the torso, the torso...

She has to collect herself, takes a gulp of water.
Emmy
Her torso was put onto the barbecue spit and a fire lit underneath it. The perp was trying to get rid of the evidence...

Langer
And your point is....?

Emmy
I have two points. One is that there was a cross cut in the chest, and the other...the other...

von Neumann
The other was....?

Emmy
The body didn’t burn. It remained intact. She was....

The tension in the room is palpable.

Emmy
She was irreducible...

Langer stares at Emmy.

Langer
I don’t believe it. Let’s see the evidence. Where did you find her?

Emmy
Finite Fields. I warn you, it isn’t pretty...

Langer
I want to see.

Emmy clicks the screen. A charred, blackened torso appears, with an unmistakable cross carved into it.

There is a loud THUMP. Langer has fainted.

The whole room suddenly bursts into applause. von Neumann stands up. Furious.

von Neumann
A circus show. A trick. It’s not enough.

The applause falters.
von Neumann (cont’d)

Before I can make an arrest I need a motive. All these brilliant theories that you’ve put forward lack common sense; they’re just not going to convince a jury.

Emmy flushes - and then jumps as there is a loud sound of a book being banged shut. It is Gauss. Einstein hits the floor.

Gauss

It is impossible to concentrate enough to read in here. Did I hear you correctly, Detective? You’re still unconvinced? Unbelievable! We have done what we were asked to do. We have found the link between the bodies... And much, much more besides.

von Neumann

But it is improbable.

Gauss

“Improbable as it is, all other explanations are more improbable still.”

von Neumann

Even if I grant you that, I still want to understand why - Why are their anatomies so similar? Do they have the same DNA? Is one modeled on the other? You don’t seem to be talking about genetics so how can the math tell you all this with no biological explanation....

Gauss holds his hand up to stop von Neumann’s flow.

Gauss

Langer outlined our responses to these questions in his presentation, detective. We have proposed two possible explanations. One from probability theory, the other from analytic combinatorics - turn to your hand-outs, section 5.1. Admittedly neither is incredibly persuasive, but that is as much as we know for now.

All the police officers rustle through to find the place in the papers in front of them.
Gauss (cont’d)
The important thing is that we now suspect that many other populations have remarkably similar anatomies. Our main investigation concerned Permutations and Integers. Emmy uncovered Polynomials in finite fields - these decompose into irreducible polynomials, and roughly one in every $N$ polynomials of degree $N$ is irreducible. We are now looking into equivalence classes of maps from a set to itself, and several other populations. A Russian forensic mathematician, Detective Professor Vershik from St. Petersburg, came up with these kinds of theories a few years ago, they were briefly in vogue and then forgotten. But he was right, von Neumann, he was right. Even if he did not have all the evidence that we have, he was right. So it is up to you, Detective von Neumann, to use this information and to determine what the possibilities are. We have opened the door for you, it is up to you to decide whether to go through it.

von Neumann looks through the papers.  

Von Neumann
Well Gauss - I am still not convinced of the motive - but we have the perpetrator.

He nods at two of his men, who swiftly move to pull Langer to his feet.

Von Neumann
Sergei Langer, I am arresting you on suspicion of the murder of Permu-ta-tion, Arnie Integer and Polly No-mial. You have the right to remain silent. Anything you say, can and will be used in evidence against you in a court of law. You have the right to have an attorney present during questioning. If you cannot afford an attorney, one will be appointed for you.

Emmy
What’s going on? Langer?

Gauss
Newman?
VON NEUMANN
Yes, it is Langer. I had my suspicions when he compared the two murders to apples and i-pods - Permu-Ta-Tion had been wearing an i-pod when she was murdered, but it was missing when we found her.

EMMY
Then, how do you know that she was wearing an i-pod?

VON NEUMANN
During the initial autopsy the coroner discovered a song playing inside her head – you know – the sort of song that you just can’t get out of your head. This only happens when a victim is listening to their i-pod, but the i-pod was gone. Our first big break in the case. Then Langer mentioned apples and i-pods, our second big break. I was suspicious so I contacted my clairvoyant -

GAUSS
Newman! You use a clairvoyant?

VON NEUMANN
I think you’ll be surprised to learn, Professor, that my clairvoyant is none other than Joe Ten-Dick, now living deep in the Pyrenees, meditating on free choice, determinism and the existence of evil - and when he is presented with seeking out evil, he sends me these.....

von Neumann pulls out several pieces of large art paper with swathes of color and pattern on them.

EMMY
They look like childrens’ drawings!

LANGER
They show nothing specific.

VON NEUMANN
But it was enough to give me the lead that I needed. You of all people would surely agree that it is enough. That and....

He pulls out a plastic evidence bag with an i-pod in it.
von Neumann (cont’d.)

....this. Found in your closet when we searched your apartment whilst you showed the camera crew around the morgue. Her DNA is all over it.

Langer straightens up and beckons to the camera crew.

Langer
Alright, I'll confess - I have no interest in remaining anonymous any longer. My work is done.

Gauss
Langer - you - Why...?

Langer
Joe Ten-Dick taught us that there is a commonality between so many different classes of mathematical objects. I wanted to discover some underlying structure. It never appealed to me to look at what one can see on the surface, I had to go underneath. Bourbaki explained to me that to get results you cannot be afraid; that you have to dig deeper.

Gauss
But in the abstract, always in abstract terms.

Langer
That’s why I killed them. Each of them was carefully chosen to represent different populations.

Emmy is silently crying. Gauss puts his arm around her. Langer puts his fist to his scar.

Langer (cont’d.)
I needed to dissect them to understand what was inside each of them. I learnt my lessons well, Professor Gauss – I made copious notes on everything which I know you will want to study.

He pulls a sheaf of notes from his briefcase and hands them to Gauss, who disdainfully rips them into shreds. Langer whimpers in disbelief.
Langer
Those are all my results - all my findings. You don’t understand. You have destroyed all that knowledge that could be of use to mankind.

Gauss
At what cost, Langer?

Langer
At great personal cost. You wouldn’t understand. I’ve been pursuing this research for years, I’ve looked deep into many different subjects.

Emmy
I don’t understand - I thought Joe Ten-Dick is the clairvoyant.

Gauss
Joe Ten-Dick was one of the greatest of all mathematical forensic scientists, but his viewpoint was too far ahead of his time. He disappeared into the mathematical wilderness as soon as his theories began to be “used”, I could never understand why. But now I see he would be horrified to learn about the terrible things that have been done in his name, the appropriation of his ideas for such inappropriate ends.

Langer
When I heard that Professor Gauss was on the case I decided to sign up for classes to follow his every movement. And I began to see that he could go deeper with these classical unmotivated investigative techniques than I had imagined possible. Professor Gauss is so very clever but he never penetrated a subject like Joe Ten-Dick.

Von Neumann
That’s enough of that. I don’t want to hear any more of this nonsense. Take him away and lock him up. Now

Langer is led away by the two Police Officers.
INT. LOBBY, BRONX INSTITUTE FOR MATH - DAY

Barry is standing talking to camera, microphone in hand.

BARRY
We are here today to hear Professor Emmy Germain’s inaugural lecture on the Political Correctness of Forensic Mathematics - detailing her spectacular discovery that the three victims, Arnie Integer, Permu Ta Tion and Polly Nomial, their structures and anatomies, are more or less identical underneath the skin.

DISSOLVE TO: INT. HI-TECH CLASSROOM, BRONX INSTITUTE FOR MATH - DAY

Gauss sits at the back of the auditorium, dozing. Emmy stands at the podium, in full academic garb, with slides of various corpses - all different shapes, sizes and colors - projected behind her.

EMMY
So, to conclude, we see that these three mathematical populations, permutations, integers and polynomials in finite fields, are the same underneath the skin if you ignore the peripherals. Their structures, their anatomies, even to the most minute details seem to be more-or-less identical, one no better than or no worse than the other. And if we look at a vast array of mathematical populations then we see that there are a small number of different possible anatomy types, and within each possibility there are many mathematical populations whose anatomies are far more similar than the differences that distinguish them. Thank you.

The whole auditorium rises to give Emmy a standing ovation.

FADE OUT AS CREDITS ROLL:

Written by

JENNIFER AND ANDREW GRANVILLE

Adapted from the article:

“Anatomy of Integers and Permutations”

by ANDREW GRANVILLE.