24. *(3 points.)* If p is an odd prime, then $0^2, 1^2, 2^2, \ldots, ((p-1)/2)^2$ are obviously all squares modulo p. There are no other squares modulo p, for given any $a \bmod p$, $a \equiv \pm b \bmod p$ for some $b \in \{0, 1, \ldots, (p-1)/2\}$, hence $a^2 \equiv b^2 \bmod p$. Also, the b^2 are all distinct modulo p, for if $i^2 \equiv j^2 \bmod p$ then $(i+j)(i-j) \equiv 0 \bmod p$, that is $i \equiv j \bmod p$ or $i \equiv -j \bmod p$. Thus, apart from 0, there are precisely $(p-1)/2$ quadratic residues modulo p, and there must therefore be precisely $(p-1)/2$ quadratic non-residues modulo p. Hence

$$\sum_{j=1}^{p-1} \left(\frac{j}{p}\right) = \frac{p-1}{2} - \frac{p-1}{2} = 0.$$

Alternatively, choose an $a \bmod p$ such that $(a/p) = -1$. (Such an $a \bmod p$ exists because there are at most $(p+1)/2$ squares modulo p, as we noted earlier.) Since $(a,p) = 1$, $aj \bmod p$ runs over all nonzero residue classes modulo p as j does. Therefore

$$\sum_{j=1}^{p-1} \left(\frac{j}{p}\right) = \sum_{j=1}^{p-1} \left(\frac{aj}{p}\right) = \sum_{j=1}^{p-1} \left(\frac{a}{p}\right)\left(\frac{j}{p}\right) = \left(\frac{a}{p}\right) \sum_{j=1}^{p-1} \left(\frac{j}{p}\right) = -\sum_{j=1}^{p-1} \left(\frac{j}{p}\right).$$

Hence

$$\sum_{j=1}^{p-1} \left(\frac{j}{p}\right) = 0.$$

x. *(2 points.)* We use a list of primes up to 1009 to check that $p_k < 2p_{k-1}$ for every odd prime $p_k \leq 1009$. In fact 509 is prime so this must be true of every prime between 510 and 1018; 257 is prime so this must be true of every prime between 258 and 514; 131 is prime so this must be true of every prime between 132 and 262; 67, 37, 19, 11, 7, 5, 3, 2 are all prime. If n is any integer between 2 and 1008, then $p_{k-1} \leq n < p_k$ for some odd prime $p_k < 1009$, hence $n < p_k < 2p_{k-1} \leq 2n$.

Total: 5 points.