1. The functions \( f(x) = 4x - 4x^2 \) and \( \sin \pi x \) agree at \( x = 0, 1/2, \) and 1. Show that \( f(x) \geq \sin \pi x \) for \( 0 \leq x \leq 1. \)

2. Determine, with proof, all functions \( f \) defined on the set of integers and satisfying
\[
f(n + m) + f(n - m) = 2(f(m) + f(n))
\]
for all \( n \) and \( m. \)

3. Let \( f(x) = \frac{x^3e^{x^2}}{(1-x^2)^2}. \) Find \( f^{(2012)}(0). \) (Here \( f^{(n)} \) denotes the \( n \)th derivative of \( f. \))

4. Let
\[
f(x) = \frac{1}{1 - x}.
\]
Let \( f_1(x) = f(x) \) and for each \( n = 2, 3, \ldots, \) let \( f_n(x) = f(f_{n-1}(x)). \) What is the value of \( f_{2012}(2012)? \)

5. Evaluate \( \int_0^\frac{\pi}{2} \ln(\sin x) \, dx. \)

6. Let
\[
I_\alpha = \int_0^\infty \frac{dx}{x^\alpha (1+x)}, \quad 0 < \alpha < 1.
\]
Find the choice of \( \alpha \) that minimizes \( I_\alpha. \) Explain.

7. Let \( f \) be a continuous, decreasing function on \([0,1]\). Show that
\[
\int_0^1 f(x)(1-2x) \, dx \geq 0.
\]

8. Evaluate
\[
\int_0^\infty \frac{\arctan(\pi x) - \arctan(x)}{x} \, dx.
\]

9. Let \( T \) be the triangle with vertices \((0,0),(a,0),\) and \((0,a).\) Find
\[
\lim_{a \to \infty} a^4e^{-a^3} \int_T e^{x^3+y^3} \, dxdy.
\]