

MATH 103. SECTION 205. QUIZ 2A
MARCH 1ST, 2007

NAME:

STUDENT NUMBER:

Calculators and other electronic devices are neither allowed nor required for this test.
This exam has 2 pages.

Compute the following integrals:

1. $\int \frac{dx}{x^2 + x - 2}$

Solution: $\int \frac{dx}{x^2 + x - 2} = \int \frac{dx}{(x-1)(x+2)}$.

We will solve this problem using partial fractions

$$\frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2} = \frac{A(x+2) + B(x-1)}{(x-1)(x+2)}$$

$$\begin{cases} A + B = 0 \\ 2A - B = 1 \end{cases} \cdot \text{From first equation } A = -B, \text{ adding both equations, } 3A = 1.$$

Therefore, $A = \frac{1}{3}$ and $B = -\frac{1}{3}$.

(We can also solve it by plugging $x = -2$ and $x = 1$ in $A(x+2) + B(x-1) = 1$.)

$$\int \frac{dx}{x^2 + x - 2} = \int \left(\frac{1}{3(x-1)} - \frac{1}{3(x+2)} \right) dx = \frac{1}{3} \ln|x-1| - \frac{1}{3} \ln|x+2| + C$$

2. $\int x \sin(x) dx$

Solution: We solve this problem using integration by parts. Let $u = x$ and $dv = \sin(x)$. Then $du = dx$ and a possible $v = -\cos(x)$.

$$\int x \sin(x) dx = x(-\cos(x)) - \int (-\cos(x)) dx = -x \cos(x) + \int \cos(x) dx = -x \cos(x) + \sin(x) + C$$

3. $\int \frac{\ln(3x)}{x} dx$

Solution: We solve this problem by using substitution $u = \ln(3x)$. Then $du = \frac{dx}{x}$.

$$\int \frac{\ln(3x)}{x} dx = \int u du = \frac{u^2}{2} + C = \frac{\ln^2(3x)}{2} + C$$

4. $\int_0^2 \sqrt{4-x^2} dx$ (hint: $x = 2 \sin u$, $\int_0^{\frac{\pi}{2}} \cos^2 y dy = \frac{\pi}{4}$)

Solution: We do the substitution $x = 2 \sin u$. Then $dx = 2 \cos u du$

$$\int_0^2 \sqrt{4-x^2} dx = \int_0^{\frac{\pi}{2}} \sqrt{4-4\sin^2 u} 2 \cos u du = \int_0^{\frac{\pi}{2}} 4 \cos^2 u du = \frac{4\pi}{4} = \pi$$

The problem can be also solved by observing that we are computing one fourth of the area of the circle of radius 2.