MATH 103. SECTION 205. QUIZ 1A

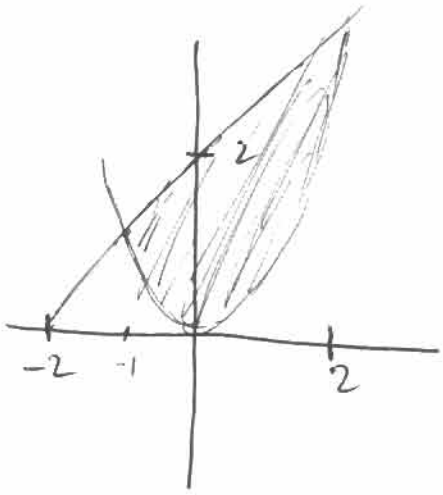
FEBRUARY 1ST, 2007

NAME:
STUDENT NUMBER:

1. Find the area enclosed between the graphs of x^2 and $x+2$ using integrals and the Fundamental Theorem of Calculus.

Solution: We look for the intersection of graphs. We need to solve

$$x^2 = x + 2.$$



So,

$$x^2 - x - 2 = 0$$

$$(x+1)(x-2) = 0$$

Therefore, the graphs intersect at $x = -1$ and $x = 2$.
We need to compute

$$\int_{-1}^{2} (x + 2 - x^2)\, dx = \left(\frac{x^2}{2} + 2x - \frac{x^3}{3}\right)\Big|_{-1}^{2}$$

$$= \left(\frac{4}{2} + 4 - \frac{8}{3}\right) - \left(\frac{1}{2} - 2 + \frac{1}{3}\right) = \frac{9}{2}$$

2. Compute the following integrals using the Fundamental Theorem of Calculus.

a.

$$\int_0^{\frac{\pi}{2}} \cos(5x)\, dx = \frac{\sin(5x)}{5}\Big|_0^{\frac{\pi}{2}} = \frac{1}{5} - \frac{0}{5} = \frac{1}{5}$$

b.

$$\int_{-2}^{1} 2e^{3x}\, dx = 2\frac{e^{3x}}{3}\Big|_{-2}^{1} = \frac{2e^3}{3} - \frac{2e^{-6}}{3} = \frac{2(e^3 - e^{-6})}{3}$$

c.

$$\int_0^1 x^2(1-x)\, dx = \int_0^1 (x^2 - x^3)\, dx = \frac{x^3}{3} - \frac{x^4}{4}\Big|_0^1 = \frac{1}{3} - \frac{1}{4} = \frac{1}{12}$$

MATH 103. SECTION 205. QUIZ 1B

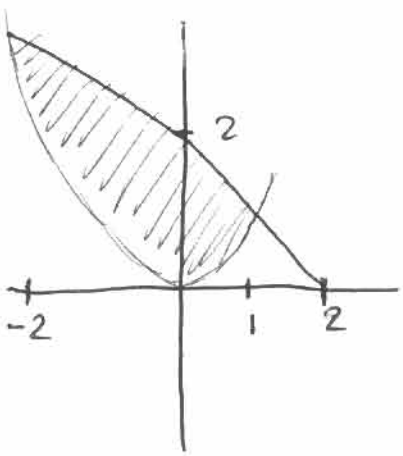
FEBRUARY 1ST, 2007

NAME:
STUDENT NUMBER:

1. Find the area enclosed between the graphs of x^2 and $2-x$ using integrals and the Fundamental Theorem of Calculus.

Solution: We look for the intersection of graphs. We need to solve

$$x^2 = 2 - x.$$



So

$$x^2 + x - 2 = 0$$

$$(x-1)(x+2) = 0$$

Therefore, the graphs intersect at $x = 1$ and $x = -2$.
We need to compute

$$\int_{-2}^{1} (2 - x - x^2)\, dx = \left(2x - \frac{x^2}{2} - \frac{x^3}{3}\right)\Big|_{-2}^{1}$$

$$= \left(2 - \frac{1}{2} - \frac{1}{3}\right) - \left(-4 - \frac{4}{2} + \frac{8}{3}\right) = \frac{9}{2}$$

2. Compute the following integrals using the Fundamental Theorem of Calculus.

a.

$$\int_0^{\frac{\pi}{2}} \cos(9x)\, dx = \frac{\sin(9x)}{9}\Big|_0^{\frac{\pi}{2}} = \frac{1}{9} - \frac{0}{9} = \frac{1}{9}$$

b.

$$\int_{-1}^{2} 2e^{3x}\, dx = 2\frac{e^{3x}}{3}\Big|_{-1}^{2} = \frac{2e^6}{3} - \frac{2e^{-3}}{3} = \frac{2(e^6 - e^{-3})}{3}$$

c.

$$\int_0^1 x(1-x^2)\, dx = \int_0^1 (x - x^3)\, dx = \frac{x^2}{2} - \frac{x^4}{4}\Big|_0^1 = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$