

## MATH 2<sup>9</sup> Algebraic Number Theory

**Instructor:** Matilde N. Lalín, CAB 621, mlalin@math.ualberta.ca

**General Description:** The fundamental theorem of arithmetic says that every positive integer number can be written as product of prime numbers in an essentially unique way.

More generally, it makes sense to speak of factorization in integral domains, and more precisely in rings in number fields (finite extensions of  $\mathbb{Q}$ ). A natural question is, do all the rings in numbers fields satisfy unique factorization?

In the late 1840s, Kummer discovered a “proof” of Fermat’s last theorem, the assertion that  $X^n + Y^n = Z^n$  has no nontrivial solutions for  $n > 2$ . Kummer’s proof depended on unique factorization in the rings  $\mathbb{Z}[\omega]$ , where  $\omega$  is a root of unity. (Un?)fortunately, not every number field has unique factorization. Kummer discovered that his proof wouldn’t always work. These considerations gave rise to what is known as algebraic number theory.

Algebra Number Theory deals with number fields, their ideals, their rings of integers, their units.

**Book:** Some recommended books (all are optional): “A Brief Guide to Algebraic Number Theory”, by H.P.F. Swinnerton-Dyer; “Number Fields”, by D. A. Marcus; “Algebraic Number Theory”, by S. Lang.

**Prerequisites:** Some basics of group theory and Galois theory. Feel free to ask the instructor if you have any questions.

**Tentative Syllabus:** Number Fields and Ideals (Dedekind domains, unique factorization of ideals, ideal class group, geometry of numbers and finiteness of the class number, the unit theorem), Valuations (valuations and completions, Idèles and Adèles), Special Fields (Quadratic, Cyclotomic, etc, applications to Fermat’s last theorem), Analytic Methods (Zeta and L-functions, analytic continuation, density theorems).