MATH 512 A1 *Algebraic Number Theory*

Lectures: September 3 - December 3
MWF 12:00 - 12:50 CAB 657
No classes on Monday October 13
Special arrangements for the weeks November 2-7 and 23-26

Instructor: Matilde N. Lalín
CAB 621, office hours Mondays 1:00 - 2:00, Wednesdays 11:00 - 12:00, and by appointment (if the door of my office is open you're welcome to come in)
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Textbook: "Algebraic Number Theory", by J.S. Milne
available at http://www.jmilne.org/math/CourseNotes/math676.html
"A Brief Guide to Algebraic Number Theory", by H.P.F. Swinnerton-Dyer
"Number Fields", by D. A. Marcus
"Algebraic Number Theory", by S. Lang.
Books are optional

Assignments: They will be posted on the website. They will be due in class as follows:
September 17, October 1, October 15, October 29, November 12, December 3.
Late assignments will not be accepted.

Weights: Homework assignments will have the same weight.
The worst of the six marks will be dropped. There will be no final exam.

Grading: Based on a combination of absolute measures and distribution.

General Description: The fundamental theorem of arithmetic says that every positive integer number can be written as product of prime numbers in an essentially unique way.

More generally, it makes sense to speak of factorization in integral domains, and more precisely in rings in number fields (finite extensions of $\mathbb{Q}$). A natural question is, do all the rings in numbers fields satisfy unique factorization?

In the late 1840s, Kummer discovered a "proof" of Fermat's last theorem, the assertion that $X^n + Y^n = Z^n$ has no nontrivial solutions for $n > 2$. Kummer's proof depended on unique factorization in the rings $\mathbb{Z}[\omega]$, where ω is a root of unity. (Un?)fortunately, not every number field has unique factorization. Kummer discovered that his proof wouldn't always work. These considerations gave rise to what is known as algebraic number theory.

Algebra Number Theory deals with number fields, their ideals, their rings of integers, their units.

Syllabus: We will start with Milne's notes and (time permiting) cover some additional topics

1. Rings of integers: integral elements, norms and traces, discriminats
2. Dedekind Domains: unique factorization of ideals, ideal class group, ramification
3. The Finiteness of the Class Number: norms of ideals, geometry of numbers
4. The Unit Theorem
5. Valuations: nonarchimedean valuations, equivalent valuations, discrete valuations, Idèles and Adèles
6. Special Fields: Quadratic, Cyclotomic, etc
7. Analytic Methods: Zeta and L-functions, analytic continuation, class number formula, density theorems

Academic Integrity: The University of Alberta is committed to the highest standards of academic integrity and honesty. Students are expected to be familiar with these standards regarding academic honesty and to uphold the policies of the University in this respect. Students are particularly urged to familiarize themselves with the provisions of the Code of Student Behaviour (online at `www.ualberta.ca/secretariat/appeals.htm`) and avoid any behaviour which could potentially result in suspicions of cheating, plagiarism, misrepresentation of facts and/or participation in an offence. Academic dishonesty is a serious offence and can result in suspension or expulsion from the University.

Policy about course outlines can be found in section 23.4(2) of the University Calendar.

Students with Disabilities: Students who require accommodation in this course due to a disability are advised to discuss their needs with Specialized Support & Disability Services (2-800 Students Union Building).

Disclaimer: Any typographical errors in this Course Outline are subject to change and will be announced in class.