Math 228 (R1) Midterm

March 3rd, 2009

Instructor: Matilde N. Lalín

NAME:

SID:

1. This exam consists of 8 pages, including this one.
2. No books, notes, or electronic devices are allowed.
3. Ensure that **your *full* name and student number** appear on this page.
4. Read the questions carefully before starting to work.
5. **Justify your answers. Show all your work**
6. Continue on the back of the page if you run out of space.

Question:	1	2	3	4	5	6	7	Total
Points:	15	15	15	15	15	15	15	105
Score:								

1. (15 points) Find all the solutions $x \in \mathbb{Z}_{35}$ of the equation $20x = 15$.

2. (15 points) Solve the following system

$$x \equiv 3 \pmod{7}$$

$$x \equiv 1 \pmod{9}$$

3. (15 points) Prove by induction that

$$[7]^n = [6n+1] \quad \text{in } \mathbb{Z}_9.$$

4. Define a relation on the vector space $\mathbb{R}^2 = \mathbb{R} \times \mathbb{R} = \{(a, b) | a, b \in \mathbb{R}\}$ by the following

$$(a, b) \sim (c, d) \Leftrightarrow a + d = b + c$$

(a) (6 points) Find three vectors that are equivalent to $(2, -3)$

(b) (9 points) Show that $\sim$ defines an equivalence relation in $\mathbb{R}^2$.

5. (15 points) Show that the subset $S = \{0, 2, 4, 6, 8\}$ of $\mathbb{Z}_{10}$ is a subring. Does S have an identity?

6. The following are the addition table and the multiplication table for a ring with three elements.

$+$	r	s	t
r	r	s	t
s	s	t	r
t	t	r	s

$\cdot$	r	s	t
r	r	r	r
s	r	t	s
t	r	s	t

Answer the following questions (with justification):

(a) (3 points) What is the additive identity?

(b) (3 points) What is the additive inverse of s?

(c) (3 points) What is the additive inverse of t?

(d) (6 points) Is this a commutative ring? Does it have a multiplicative identity?

7. (15 points) Let $a, b \in \mathbb{Z}$. Prove that $(a, b) | (a + b, a - b)$.