

Midterm Examination  
Math 228 (Q1) - Winter Term

Instructor: H. Brungs

Date: Wednesday, March 2, 2005

Instructions: Justify your answers.

In Class

1. (a) Determine among the elements in  $S = \{0, 1, 2, \dots, 35\}$  those elements  $a$  which have a multiplicative inverse in  $\mathbb{Z}_{36}$ , that is  $ax \equiv 1 \pmod{36}$  is solvable.  
(b) Find in  $S$  the inverse  $5^{-1}$  of 5.
  
2. Find all solutions modulo 165 of
  - (a)  $115x \equiv 4 \pmod{165}$ ;
  - (b)  $115x \equiv 10 \pmod{165}$ .
  
3. Let  $S = \mathbb{Z}$  and  $*$  be given by  $a * b = a + b - 7$  for  $a, b \in S$ .
  - (a) Is  $*$  an operation on  $S$ ?
  - (b) Is  $*$  associative?
  - (c) Does there exist an identity  $z$  for  $*$  on  $S$ ?
  - (d) Which elements  $a$  in  $S$  have an inverse in  $(\mathbb{Z}, *)$ ?
  
4. Consider integers  $a, b, c > 0$  with  $a^2 + b^2 = c^2$ .  
Show that always one of  $a, b$  or  $c$  is divisible by 3 by considering  $a, b, c$ ,  
 $a^2 + b^2 = c^2 \pmod{3}$ .