

Midterm Examination
Math 228 (Q1) - Winter Term

Instructor: H. Brungs

Date: Wednesday, March 2, 2005

Instructions: Justify your answers.

In Class

1. (a) Determine among the elements in $S = \{0, 1, 2, \dots, 35\}$ those elements a which have a multiplicative inverse in \mathbb{Z}_{36} , that is $ax \equiv 1 \pmod{36}$ is solvable.
(b) Find in S the inverse 5^{-1} of 5.

2. Find all solutions modulo 165 of
 - (a) $115x \equiv 4 \pmod{165}$;
 - (b) $115x \equiv 10 \pmod{165}$.

3. Let $S = \mathbb{Z}$ and $*$ be given by $a * b = a + b - 7$ for $a, b \in S$.
 - (a) Is $*$ an operation on S ?
 - (b) Is $*$ associative?
 - (c) Does there exist an identity z for $*$ on S ?
 - (d) Which elements a in S have an inverse in $(\mathbb{Z}, *)$?

4. Consider integers $a, b, c > 0$ with $a^2 + b^2 = c^2$.
Show that always one of a, b or c is divisible by 3 by considering a, b, c ,
 $a^2 + b^2 = c^2 \pmod{3}$.