Problem 6:

You are given a ring $R = \{A, B, C, D\}$ with four elements and know that there is a ringhomomorphism

$$T : \mathbb{Z} \longrightarrow R,$$

that maps 1 and 5 to A, 2 and 6 to B, 3 and 7 to C and 4 and 8 to D.

(a) Using the axioms for a ring homomorphism, find $T(9)$, $T(10)$, $T(11)$ and $T(12)$, e.g. $T(9) = T(1+8) = T(1) + T(8) = T(1) + D = T(1) + T(4) = T(5) = A$. [3]

(b) Write out the *addition* and *multiplication* table for R, e.g. $A + D = T(1) + T(4) = T(1+4) = T(5) = A$ and $A \cdot D = T(1) \cdot T(4) = T(1 \cdot 4) = T(4) = D$. [6]

(c) Give the zero-element in R. [2]

(d) Decide whether R is an integral domain and whether it is a field. [2]

Total: 13 points

Problem 8:

Write the polynomial

$$\frac{1}{6}t^5 + \frac{2}{3}t^4 - \frac{1}{2}t^3 - 3t^2 \in \mathbb{Q}[t]$$

as a product of irreducible polynomials in $\mathbb{Q}[t]$.

Total: 6 points

Problem 9:

(a) Show that the polynomial $p(t) = 21t^3 - 6t + 8 \in \mathbb{Z}[t]$ has no rational root. [5]
<u>Hint:</u> Use the Gauss-Lemma and reduction modulo 5. Don't use the Rational Root Test!!!
Can you conclude that $p(t)$ is irreducible over $\mathbb{Q}$?

(b) You are given the polynomial $q(t) = 3t^{10} + 5 \cdot f(t) \in \mathbb{Z}[t]$ with $f(t) \in \mathbb{Z}[t]$ of degree < 10 and satisfying $f(0) = 17$.

i. Give such a polynomial. [1]

ii. Show that $q(t)$ is irreducible over $\mathbb{Q}$. [4]
<u>Hint:</u> Use the *Eisenstein Criterion*.

Total: 10 points