Review of BBC's Horizon Program, "Fermat's Last Theorem"

Andrew Granville

Horizon: Fermat's Last Theorem
Written and edited by John Lynch.
Directed by Simon Singh.

John Lynch and Simon Singh have succeeded in making an informative television program about Fermat's Last Theorem, its proof, and the history of that proof, while at the same time appealing to as wide an audience as possible. They accomplish these goals by Lynch's clever editing of an extensive series of interviews with Andrew Wiles, as well as by comprehensive discussions with several of the principals in this drama: John Coates, John Conway, Nick Katz, Barry Mazur, Ken Ribet, Peter Sarnak, and Goro Shimura.

The show begins by introducing us to Andrew Wiles, a researcher overwhelmed by what he has achieved, and moves on to a description of the early history of Fermat's Last Theorem using photographs, a dramatization of Fermat scribbling his infamous marginal note, and some amusing clips from an interview with Conway, posed in front of an old copy of Diophantus's *Arithmetica*. The show then leaps forward in time to the history behind Wiles's work, specifically the conjecture by Taniyama, and a discussion of Ribet's theorem. (Ribet is filmed, evidently with some delight by the British camera crew, in the outdoor Berkeley capuccino bar where Mazur helped Ribet realize that he did have a proof of that result.) This is combined with voice-overs and graphics which try to explain enough of the mathematics for the average nonmathematician to follow what is going on; more on that later.

The focus shifts to Wiles explaining how he was "electrified" by the news of Ribet's result, leading to his decision to devote his life to proving the Taniyama-Shimura conjecture. Biographical material on Wiles then follows, highlighted by ghastly file footage from TV rock music shows of different eras to help hapless viewers get historical perspective. This is cheap editing, but fortunately such crassness is the exception here. Finally we reach the story proper of Wiles's work, beginning with an attempt to discuss the role of Galois representations (of course, Singh could not resist dramatizing Galois's infamous duel). We learn that the heart of the proof lay in proving a certain "class number formula", and we hear of Wiles's attempt to prove this using the Euler systems of Flach and Kolyvagin and his failed and later revived use of Iwasawa theory. There is Wiles's decision to include Katz as a sounding board for ideas and the drama of his announcement to a specialized conference at Cambridge, his alma mater. We then hear about the mistake made in the original proof, Wiles's struggle to repair this mistake, and finally his joy at seeing how to complete a correct proof.

The BBC Web site for this program, including a complete transcript, may be found at `http://www.bbc.co.uk/horizon/95-96/960115.html`. *The program will be broadcast in the United States in the fall of 1997. The* Notices *will announce the broadcast date when it becomes available.*

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Lynch and Singh succeed in portraying what it feels like to do research by relying on the words of these eminent researchers. Wiles's remarks are worth repeating:

> I start trying to find patterns. So I'm doing calculations which try to explain some little piece of mathematics. I'm trying to fit it in with some previous broad conceptual understanding of some branch of mathematics. Sometimes that'll involve going and looking up in a book to see how it's done there; sometimes it's a question of modifying things a bit, sometimes doing a little extra calculation; and sometimes you realize that nothing that's ever been done before is any use at all and you just have to find something that is completely new, and it's a mystery where it comes from.

Wiles comes across as a very touching hero, and one feels tremendous sympathy with him in his ups and downs. After hearing about Ribet's result, showing that Fermat's Last Theorem follows from the Taniyama conjecture, Wiles

> ...was just electrified. I knew that moment that the course of my life was changing. This meant that to prove Fermat's Last Theorem, I just had to prove the Taniyama-Shimura conjecture. From that moment that was what I was working on...I decided that I really only had time for my problem and my family. When I was concentrating very hard then I found that young children provide the best possible way to relax. Talking to young children who simply aren't interested in Fermat, at this age; they want to hear a children's story, and they're not going to let you do anything else.

This single-mindedness, mixed with a very evident gentility, makes Wiles a surprisingly sympathetic person on camera, despite his shyness when being interviewed.

The program does get to the heart of this story, telling us many interesting details. It is naturally dramatic, and a very human drama at that, which can appeal to all sorts of people. The success of this program lies in the story being told primarily by the principals, with some judicious editing, rather than in too many simplifications for the average person. Most attempts to make good television about mathematics fail dismally, because either there is too much mathematics and too little visual entertainment or vice versa, so that the content is lost in the rush to popularize. This show, though not perfect, gets the balance about right, occasionally giving a voice-over explanation with cute graphics, but otherwise editing the interviews with the mathematicians, cutting from one to the next so as to use the words of the mathematicians themselves to explain the difficult concepts.

In fact, this editing technique allows Lynch to exhibit the fact that a concept is technically difficult without explicitly saying so. For example, the interviewer asks Katz to describe what a modular form is, and Katz naturally explains that he cannot do that in one simple sentence. Cut to Sarnak, who evidently has been asked the same question; he is initially stunned by the difficulty of providing a suitable answer and then just laughs at the preposterousness of trying to do so. Cut to Conway, who gushes that, "Modular curves are wonderful, highly symmetric, beautiful, intriguing, intricate things"; and cut to Mazur, who gives a similar though less enthusiastic answer (one gets the impression that he has been interviewed so many times by now that he is too ready for such questions!). What is the message? That this is deep material that even the experts find difficult to explain. Besides, the scene with Sarnak is genuinely funny, and the scene with Conway expresses well his infatuation with the concepts of mathematics.

The most refreshing parts of this program are seeing mathematics with a human face and the accurate portrayal of mathematicians' love and respect for their subject. Presumably because the cameras stayed with each of them for some time, the interviewees were able to relax and enjoy answering some of the questions and to get emotional at times. Conway, in particular, is always romping through the mathematics, charming the viewer with his effusiveness. Wiles and Shimura, who come across more drily than the others when interviewed, also are captured on film being quite moved by events. It is touching to see how Wiles is humbled by the magnitude of his achievements and how painful the memory still is for him of the mistake being found in his original proof and the intense pressure he was under, trying to fix it in the public glare. There are several very funny moments, especially when the personalities of some of the mathematicians come through.

Princeton is a special university in that it is unusually sympathetic to giving researchers the opportunity to pursue their goals. Indeed, for many of us, several aspects of Wiles's reported behaviour are surprising. For instance, we learn that he decided (and was allowed) to teach a graduate class at Princeton in order to help clarify the main part of his proof, though without let-

ting on to his students his underlying objectives. Given the depth of the material, all of the graduate students soon stopped attending, leaving only Nick Katz, a fellow faculty member, in the audience. Wiles would surely not get tenure at most universities in this country with a teaching record like that!

The program sticks closely to one version of the story, resulting in many notable events (and mathematicians) being omitted. For example, Faltings' Theorem and his many influential works are never discussed, yet these are central to the recent leaps made in understanding the nature of elliptic curves. Amazingly, André Weil is never mentioned, even though he is not only a "parent" to this field but arguably gave the first substantial reasons for believing in the truth of the Taniyama-Shimura conjecture. Indeed, Mazur remarks, "For a while it [the Taniyama-Shimura conjecture] seemed to have been ignored because it was so ahead of its time," yet the show avoids remarking how Weil's work brought it into the mainstream.

Singh's reasons for these omissions were presumably that he wanted to keep his cast small and his story focused and to ensure a coherent program in the allotted time. This makes it good television but clearly marks the limit of how much one can include in "factual television". It is a shame, in this instance, because the program did do a memorable job of capturing the sense of "culture" in mathematics, whilst failing to highlight that culture's remarkable diversity.

We are left with thought-provoking remarks for mathematicians and nonmathematicians alike, for instance: "Almost the definition of a good mathematical problem is the mathematics it generates rather than the problem itself" (Wiles); "They look so simple, yet their complexity, especially arithmetic complexity, is immense" (Sarnak on tori); or "It's a bridge, it's more than a bridge, it's really a dictionary, a dictionary where questions, intuition, insights, theorems in the one world get translated into questions and intuitions in the other world" (Mazur on the Taniyama-Shimura conjecture).

Ultimately we share the excitement of these great mathematicians, wondering with them at the summit scaled by Wiles's work. All of them—Ribet, Mazur, Katz, and the rest—marvel at Wiles's achievement, so much so that one feels it has enriched each of them personally. Perhaps Mazur best described his feelings when hearing Wiles's famous Cambridge lecture series:

> "I'd never seen a lecture series in mathematics like that before. What was unique about those lectures were the glorious ideas, how many new ideas were presented, and the constancy of its dramatic build-up. It was suspenseful until the end."

Is there a future for mathematics on television? I believe so—more after seeing this program than beforehand. It is possible to present programs at all sorts of different levels: as information, as education, and as entertainment. Unfortunately shows produced by mathematicians often seem to be intensely dry, to have amateur production qualities, and to focus on the mathematics to the exclusion of the people involved. On the other hand, shows produced by nonmathematicians tend to oversimplify the technical material, sacrificing accuracy for glitz, since they do not trust the subject matter to hold the attention of the viewers. In this show we had to endure the occasional voice-over statement, such as "Forget impossible French riddles; elliptical doughnuts was where it was at," to explain the focus on the Taniyama-Shimura conjecture rather than on Fermat's Last Theorem, but perhaps that is fair enough when trying to distill so much into such a short timespan.

What this program does illustrate is the idea that you can trust your audience to listen to a subject, replete with technicalities, as long as you try to make some of it accessible and, more importantly, as long as you allow the people behind the mathematics to be as visible as the mathematics itself.